Time-Delayed Feedback Control Method and Unstable controllers

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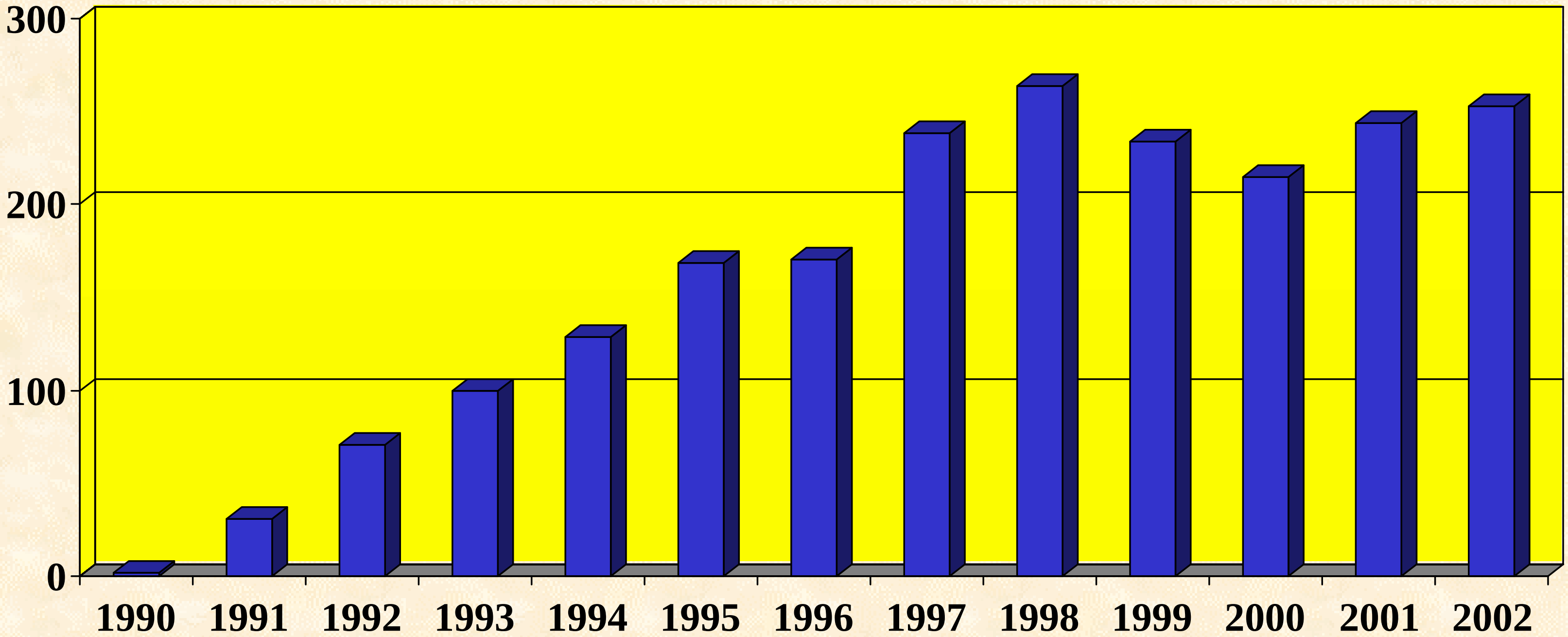
- **Introduction**
- **Delayed feedback control (DFC) method**
- **Applications and Modifications**
- **Limitation of the DFC method**
- **Unstable delayed feedback controller**
- **Using unstable controller for stabilizing steady states**
- **Conclusions**

Control of Chaos (dynamics of publications)

Pioneering paper:

- E. Ott, C. Grebogi, J.A. Yorke, Phys.Rev. Lett., **64**, 1196 (1990)



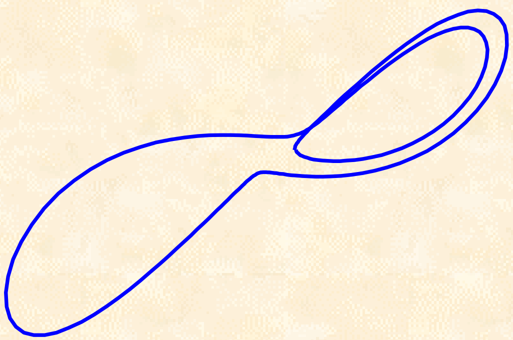
Why chaotic systems are interesting objects for control theory and applications?

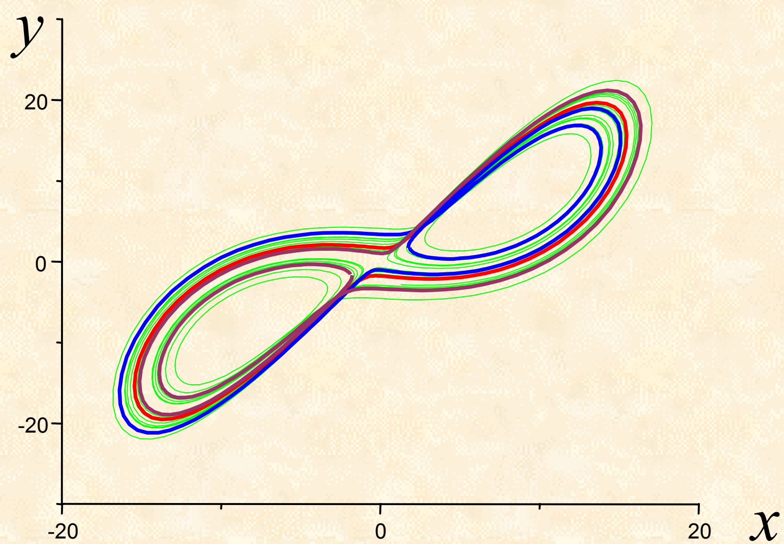
- *Chaotic systems are extremely sensitive to small perturbations (the butterfly effect)*

- *A typical strange attractor has embedded within it an infinity number of unstable periodic orbits. One can choose any of them by desire and stabilize it with only tiny perturbation.*

Unstable Periodic Orbits (Lorenz attractor)

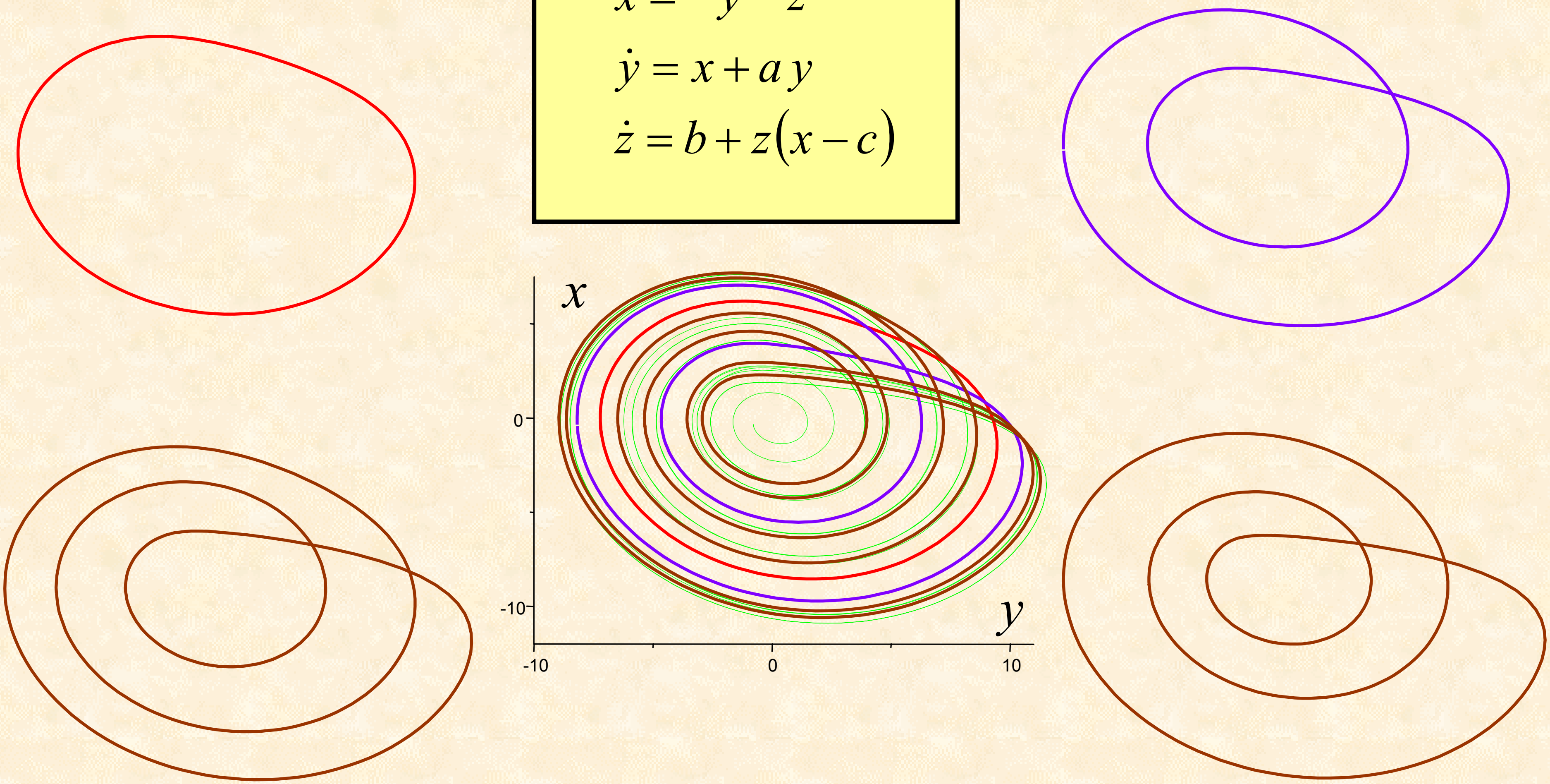
$$\dot{x} = -\sigma(y+z)$$
$$\dot{y} = rx - y - xz$$
$$\dot{z} = xy - bz$$





Unstable Periodic Orbits (Rössler attractor)

$$\dot{x} = -y - z$$
$$\dot{y} = x + a\,y$$
$$\dot{z} = b + z(x - c)$$



Delayed Feedback Control (DFC) Method

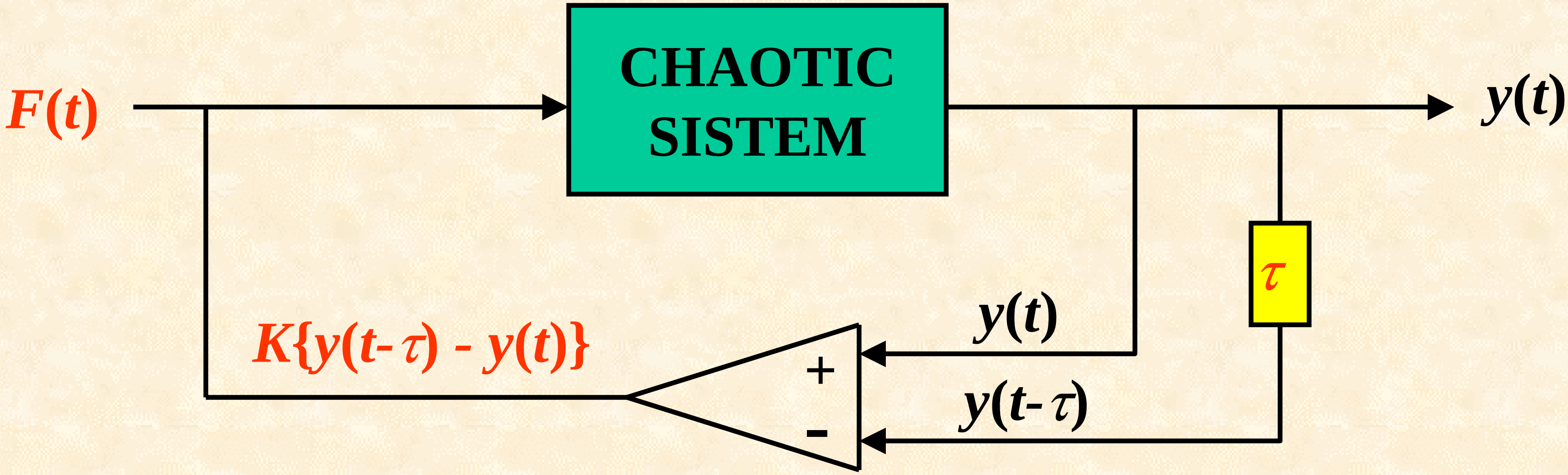
***K. Pyragas**, Phys. Lett. A **170**, 421 (1992)*

$$\frac{dy}{dt} = P(\vec{x}, y) + F(t)$$

$$\frac{d\vec{x}}{dt} = \vec{Q}(\vec{x}, y)$$

$F(t) = K\{y(t-\tau) - y(t)\}$

$\tau = T$ - ***period of unstable periodic orbit***

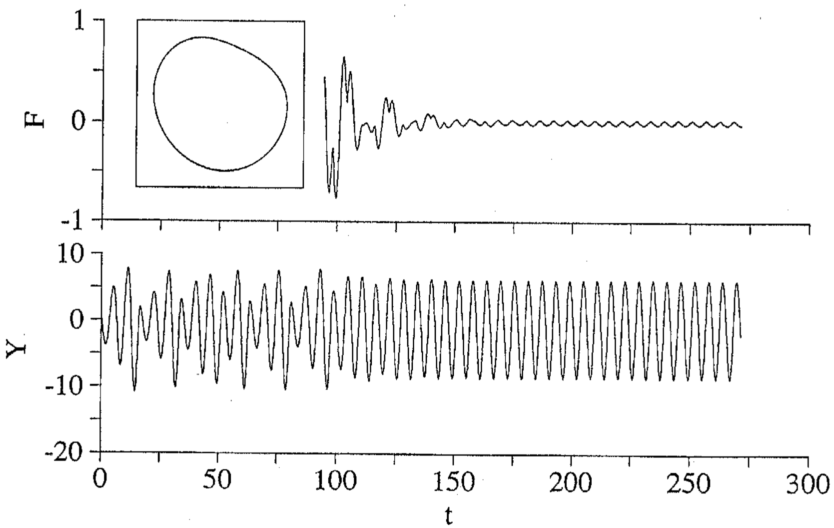


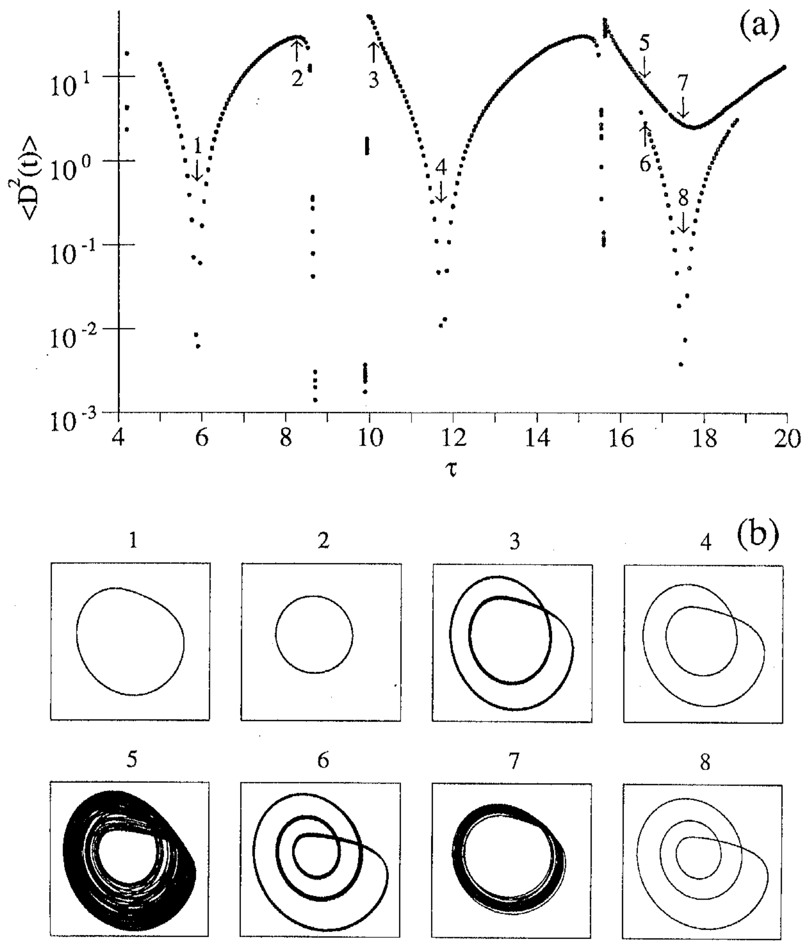
Demonstration of the DFC method (Rössler system)

$$\dot{x} = -y - z$$

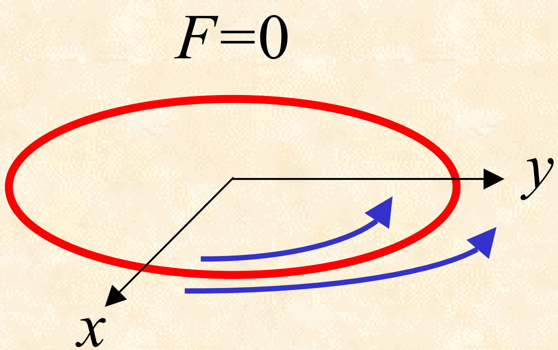
$$\dot{y} = x + a\,y \; + K\{y(t-\tau) - y(t)\}$$

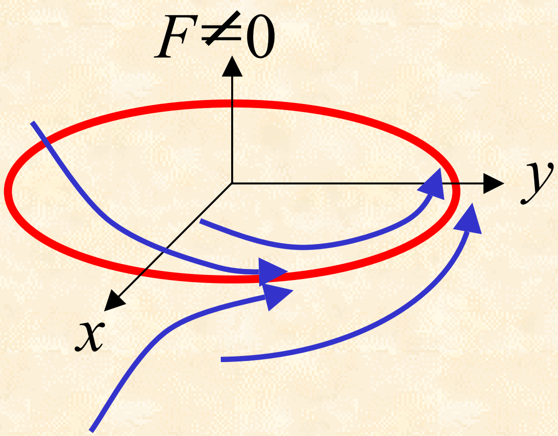
$$\dot{z} = b + z(x - c)$$





Explanation of the stabilization

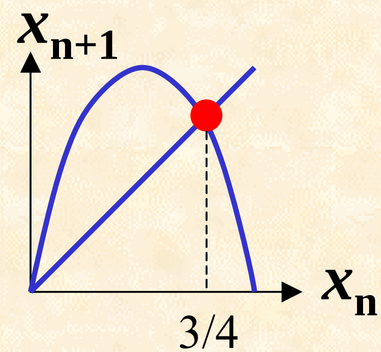


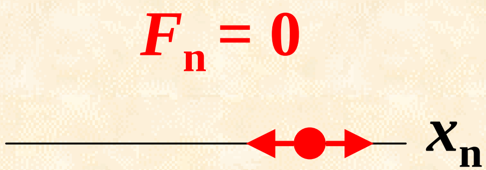


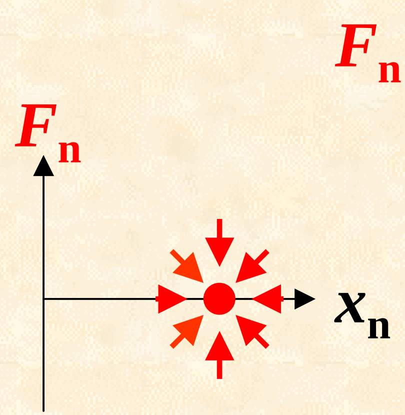
Stabilization is achieved through additional degrees of freedom introduced with the feedback perturbation!

Simple example:

$$x_{n+1} = 4x_n(1 - x_n) - KF_n\,,\quad F_n = x_n - x_{n-1}$$







Stability condition:

$$-1 < K < 1/2$$

Experimental Implementations

Electronic chaos oscillators
- Pyragas, Tamaševičius (1993)
- Gauthier et al. (1994)
- Kittel et. al. (1994)
- Celka (1994)
- Socolar et al. (1994)

Mechanic pendulums
- Hikihara, Kawagoshi (1996)
- Christini et. al. (1997)

Lasers
- Belawski et al. (1994)
- Lu, Yu, Harrison (1998)

Gas discharge systems
- Mausbach et al. (1997)
- Pierre et al. (1996)

Chemical systems
- Parmananda et al. (1999)
- Tsui, Jones (2000)

Helikopter roter blades
- Krokiewski, Faragher (2000)

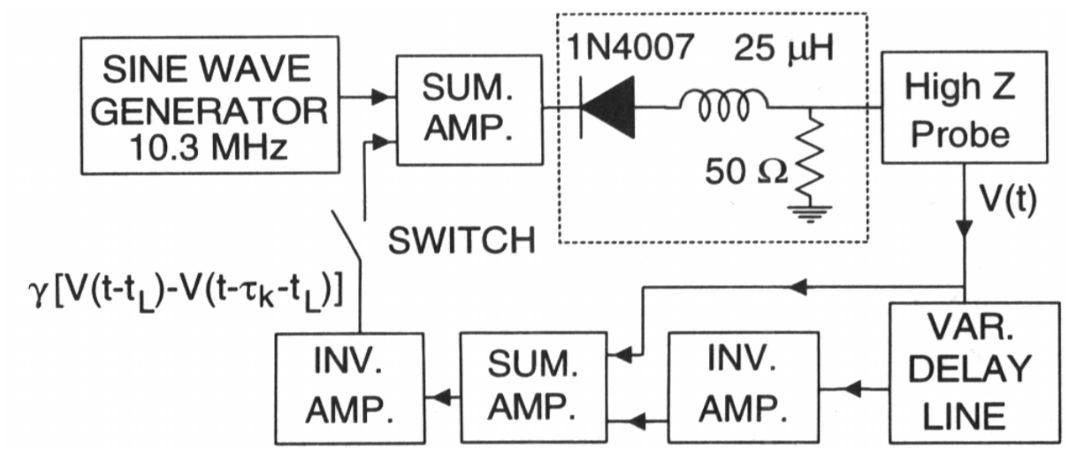
Ferromagnetic resonance
- Benner et al. (1997)

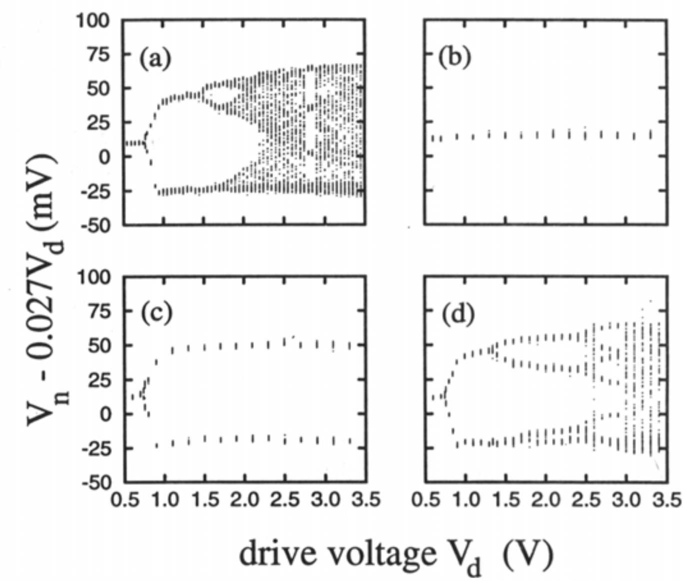
Tailor-Couette flow chaos
- Lüthje et al. (2001)

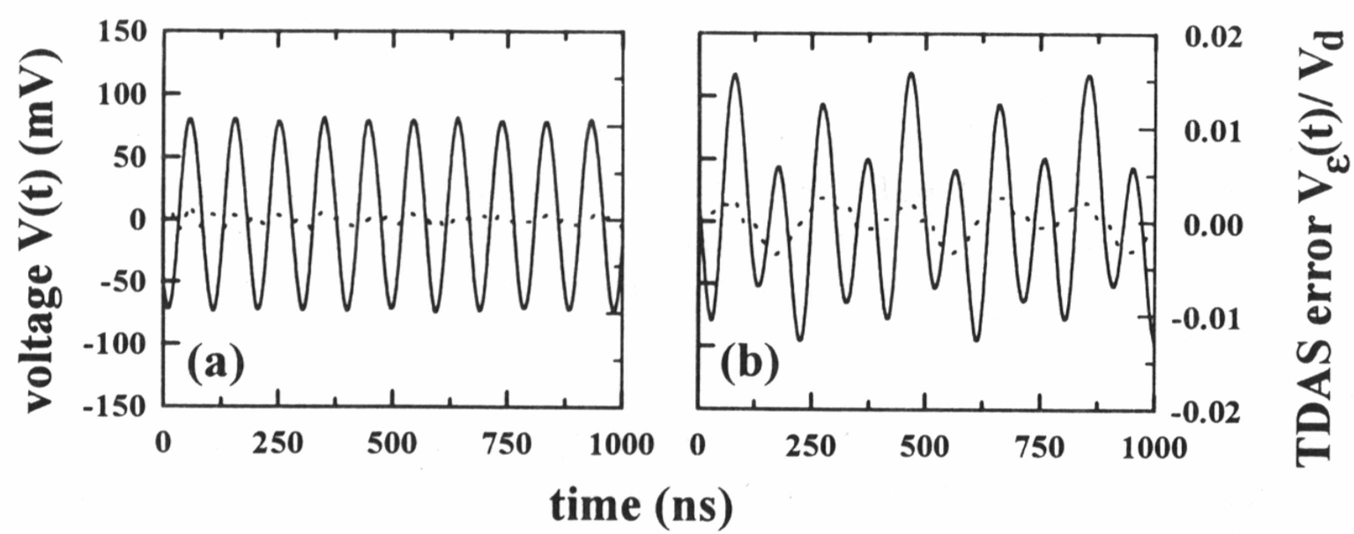
Cardiac systems
- Hall et al. (1997)
- Rappel et al. (1999)
- Skučas et al. (2001)

Controlling Chaos in a fast Diode Resonator

D.J. Gauthier et al. (1994)

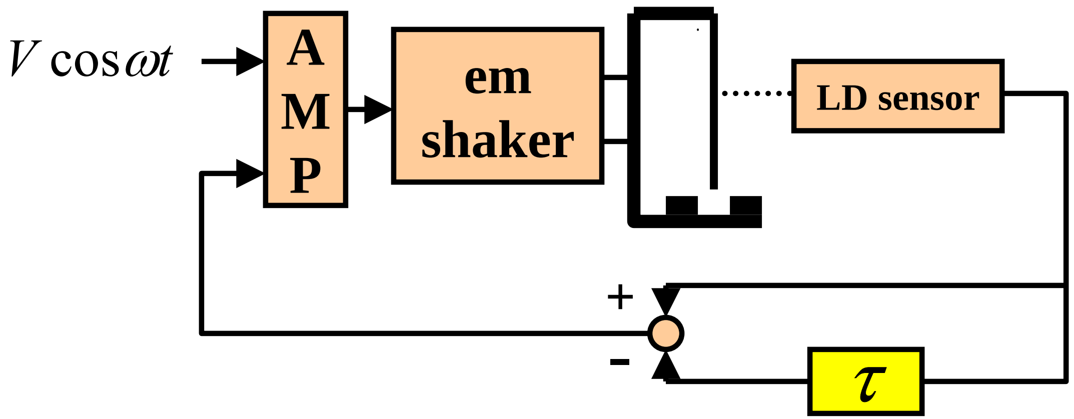


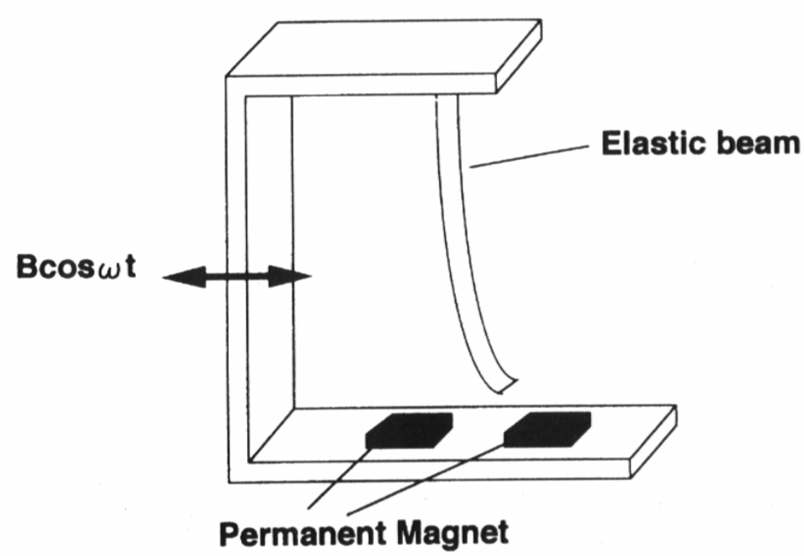


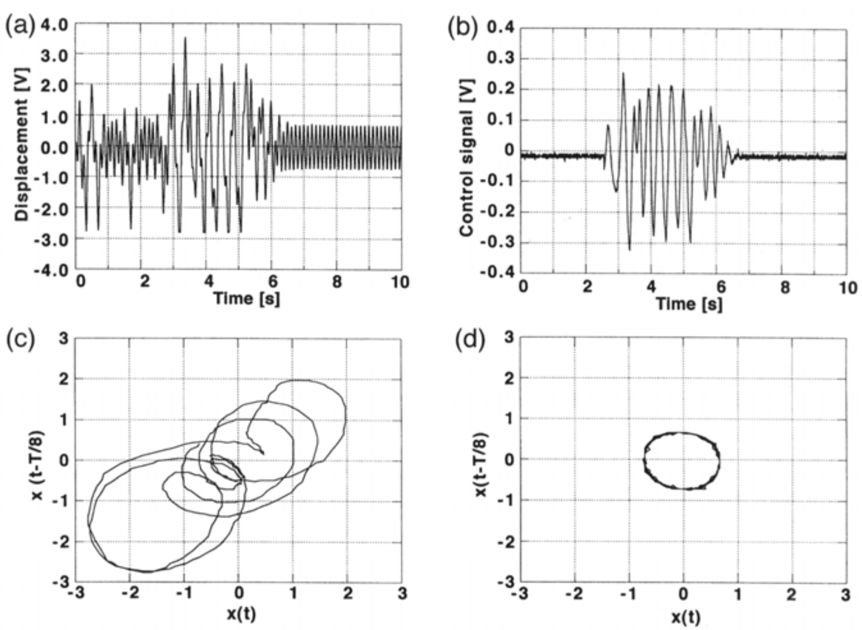


Controlling Magneto-Elastic Beam System

T. Hikihara and T. Kawagoshi (1995)

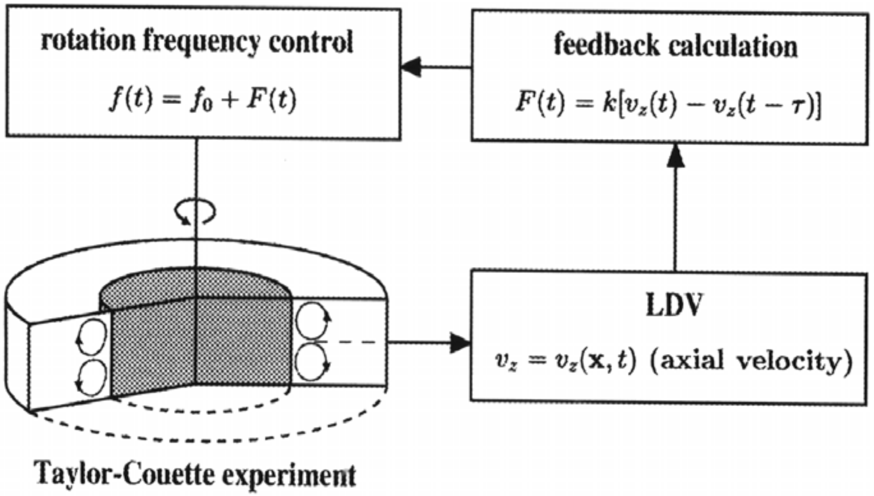






Control of Chaotic Taylor-Couette Flow

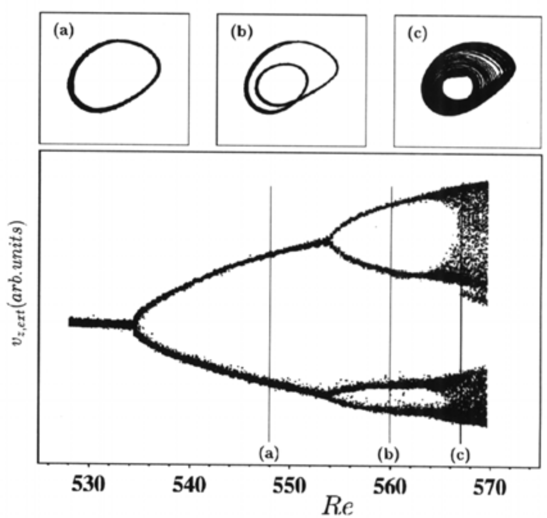
O.Lüthje, S. Wolf, and G. Pfister (2001)

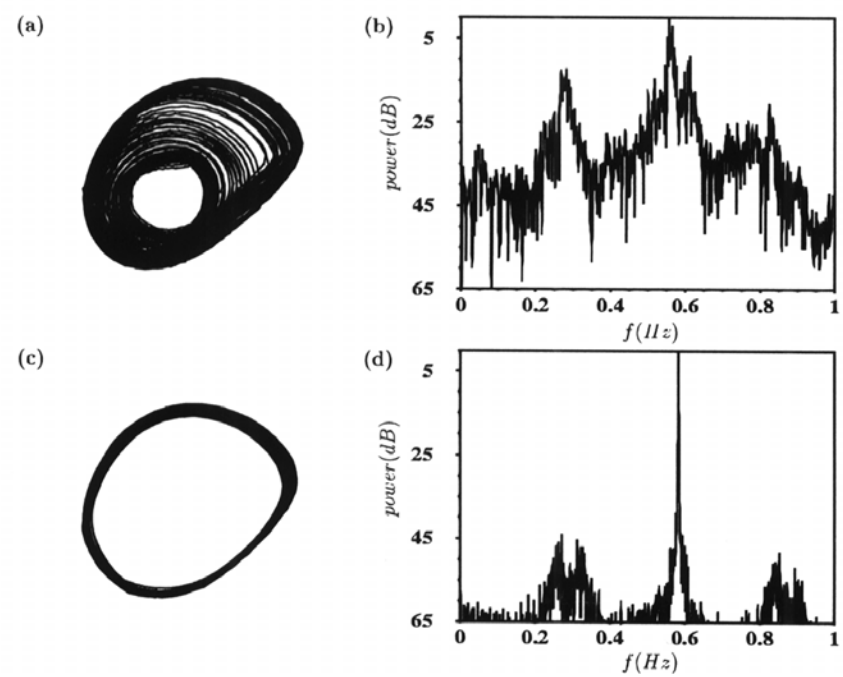


$$\mathrm{Re} = 2\pi f\,(r_o - r_i)\, r_i / \nu$$

$v_z(\mathbf{x},t)$ - output (axial velocity)

f - control parameter

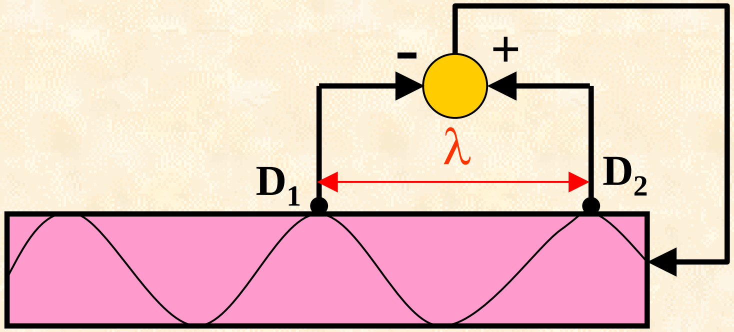




Control of Ionization Wave Chaos

Th. Mausbach et al. (1997)

The wave character of dynamics in some systems allows us to simplify the DFC algorithm by replacing the delay line with the spatially distributed detectors

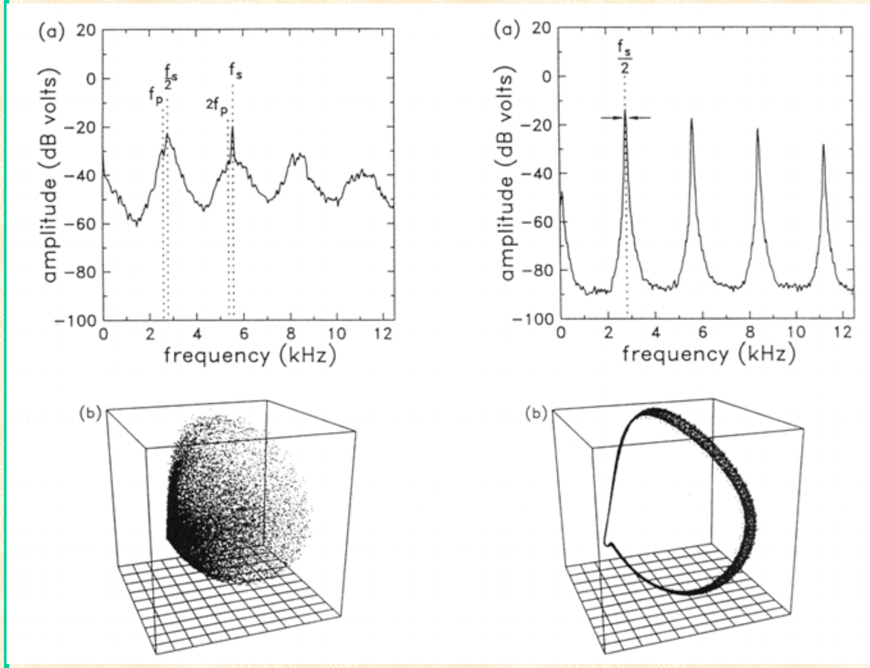


No delay line is needed !

Due to dispersion relations the delay in time can be replaced with the spatial displacement, e.g., for the plane wave $u(x,t) = Ae^{i(kx-\omega t)}$:

$$u(x,t-\tau) = u(x-\lambda,t)$$

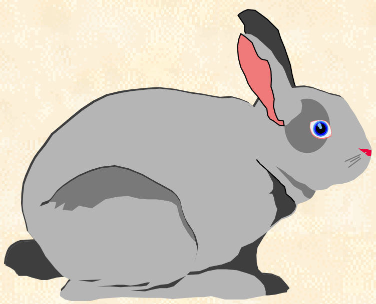
$$\lambda = \omega\tau/k$$

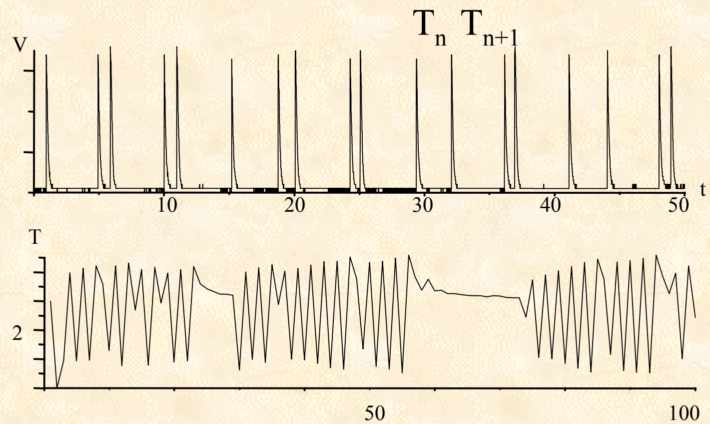


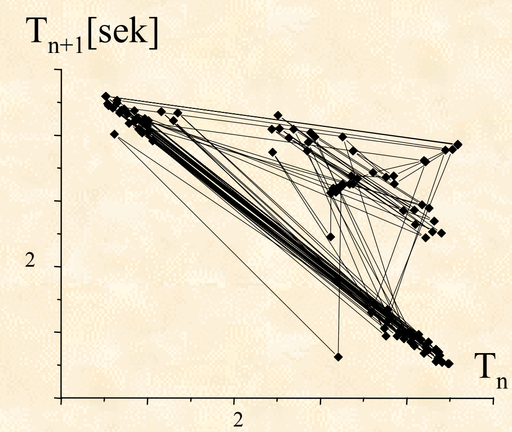
Control of isolated auricles of a rabbit heart

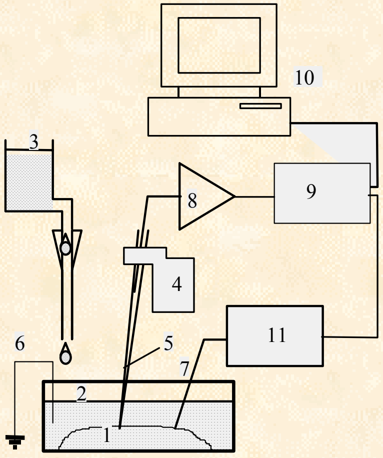
M. Skučas, I. Grigaliūnienė, V. Dzenkauskas, R. Labrencas

Cardiac institute, Kaunas , Lithuania



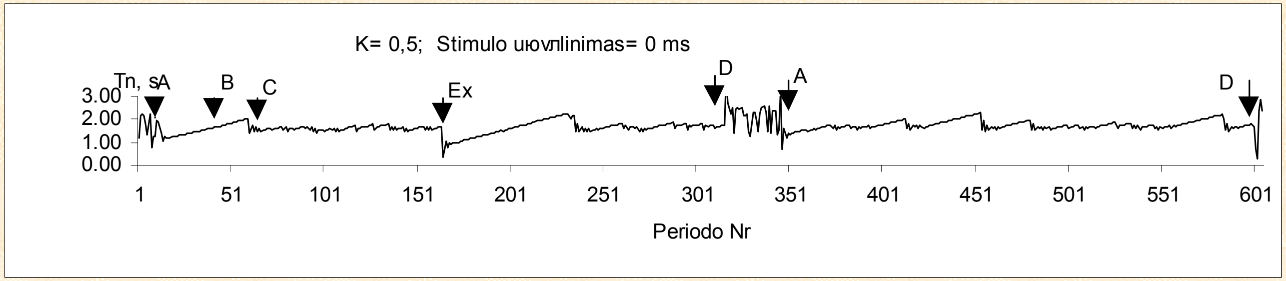






Control algorithm:

$$T_n = T_{n-1} - K\left(T_{n-1} - T_{n-2}\right)$$



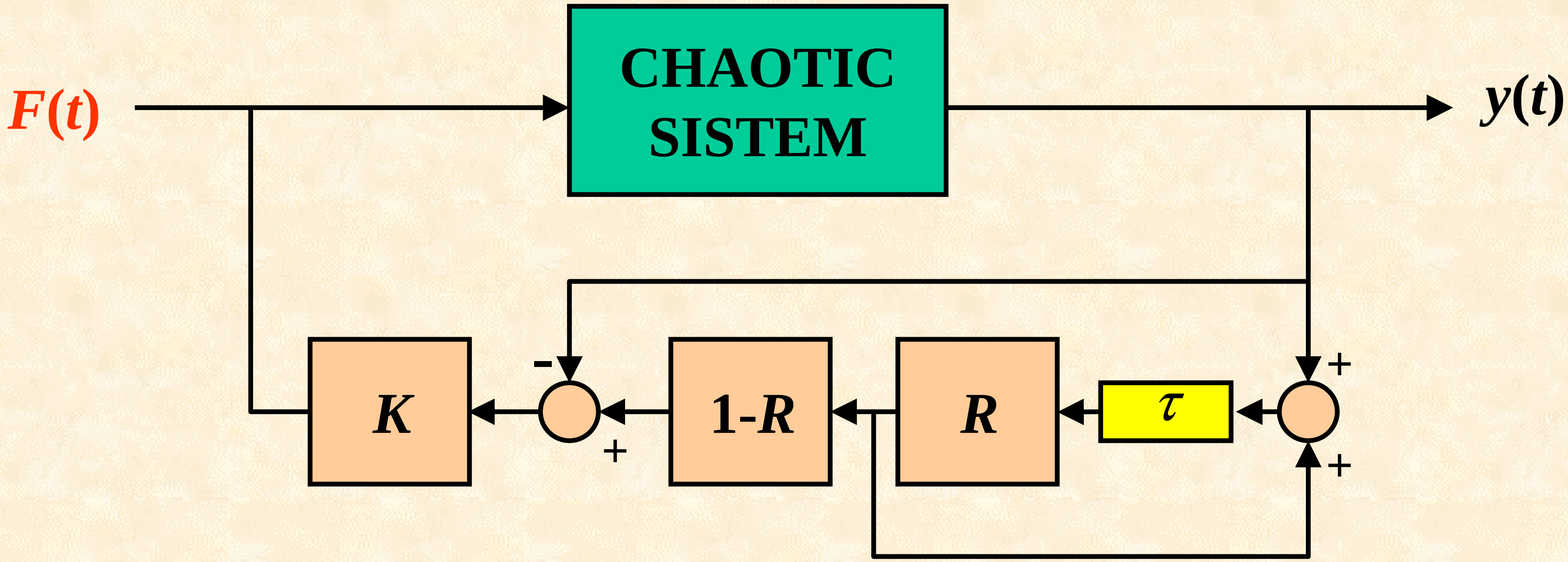
Extended Delayed Feedback Controller: **EDFC**

J. Socolar, D. Sukov, and D. Gauthier (1994)

$$F(t) = y(t) - y(t-\tau) + R\left[y(t-\tau) - y(t-2\tau)\right] + R^2\left[y(t-2\tau) - y(t-3\tau)\right] + \cdots$$

$$= y(t) - (1-R)\sum_{k=1}^{\infty} R^{k-1} y(t-k\tau),$$

$R<1$ - convergence condition

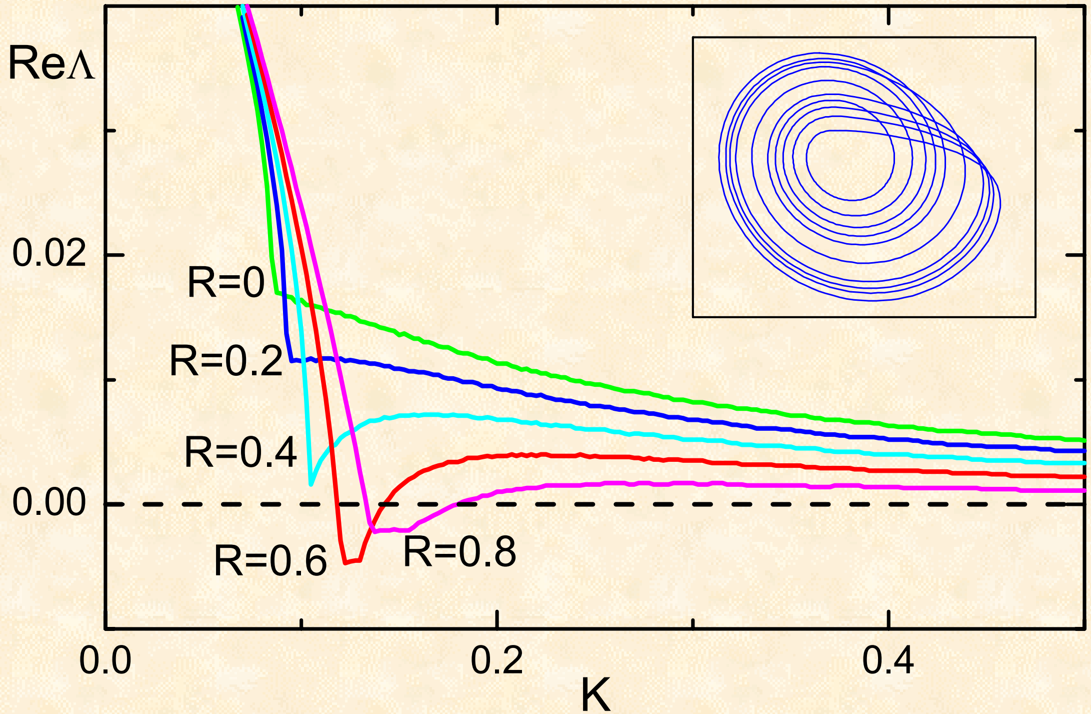


Advantages of the EDFC

Illustration for the Rössler system:

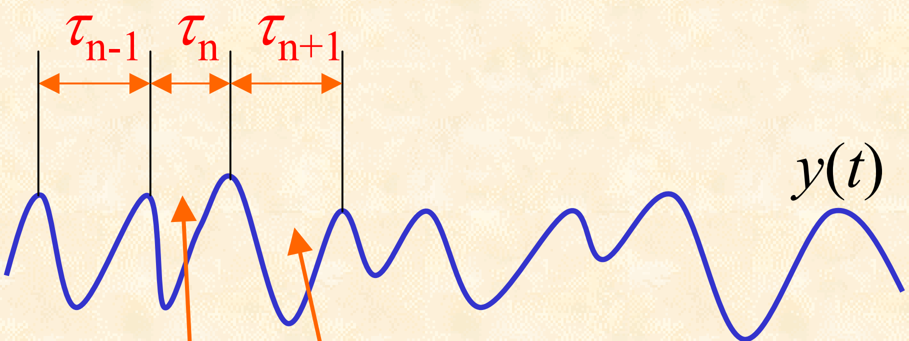
$$\dot{x} = -y - z$$
$$\dot{y} = x + ay \ - KF(t)$$
$$\dot{z} = b + z(x - c)$$

$$F(t) = y(t) - y(t-T) + R\,F(t-T)$$



Adaptive time-delayed feedback

A. Kittel, J. Parisi, K. Pyragas (1995)

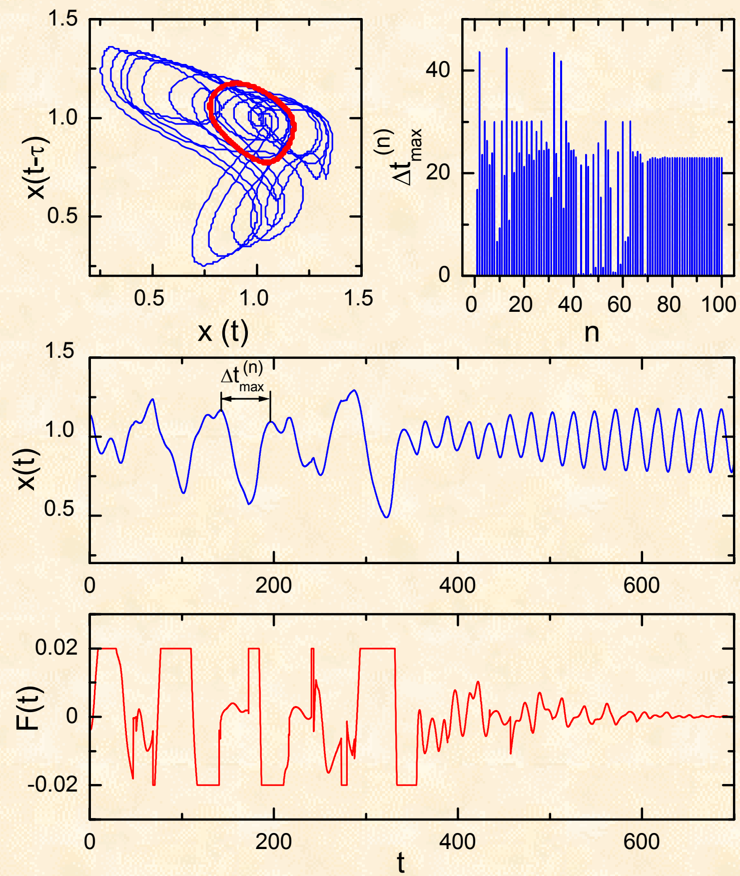


$$F_{n+1} = K[y(t) - y(t-\tau_n)]$$

$$F_n = K[y(t) - y(t-\tau_{n-1})]$$

$$n \to \infty \qquad \tau_n \to T$$

$$\frac{dy}{dt} = \frac{0.2y(t-\tau_M)}{1+y^{10}(t-\tau_M)} - 0.1y + F_n(t)$$



Limitation of the DFC method

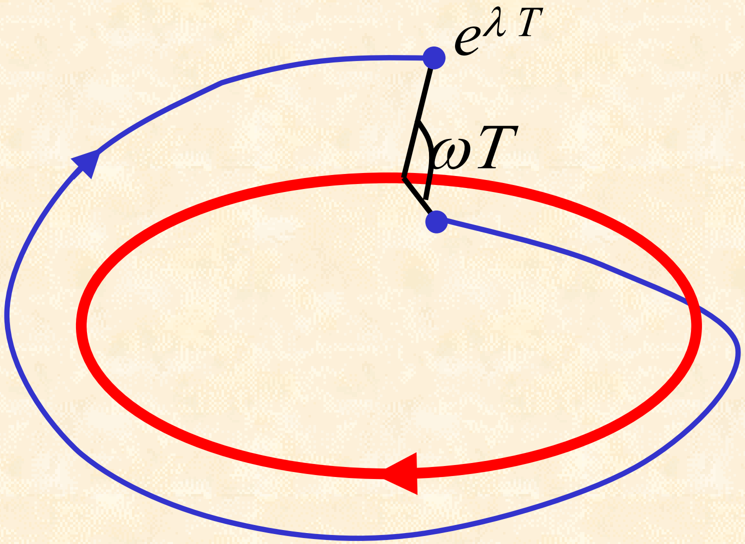
Ushio (1996), Just et al. (1997), Nakajima and Ueda (1997, 1998)

Characteristics of orbits

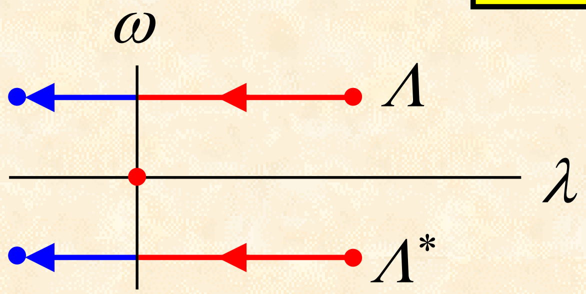
Floquet exponent or **Floquet multiplier**

$\Lambda = \lambda + i\omega$

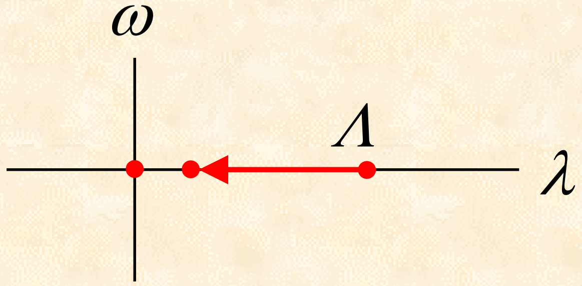
$\mu = e^{\Lambda T}$



The method works for $\omega \neq 0$



The method fails for $\omega = 0$



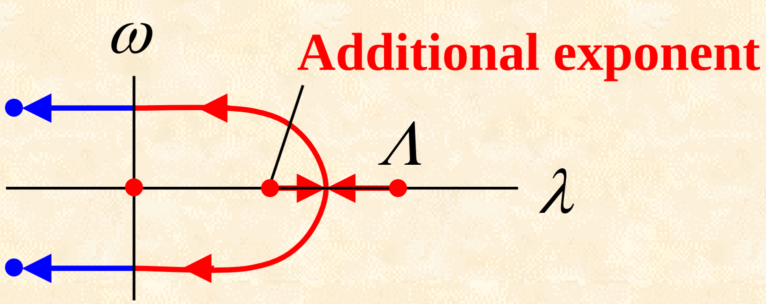
The method fails for any periodic orbits with an odd number of real Floquet multipliers >1

Unstable Delayed Feedback Controller

***K. Pyragas**, Phys. Rev. Lett.* ***86**, 2265 (2001)*

The key idea:

Artificially enlarge a set of real Floquet multipliers greater than unity to an even number by introducing into a feedback loop an unstable degree of freedom



Extended Delayed Feedback Controller for R>1
(Simple Discrete Time System)

Unstable system:

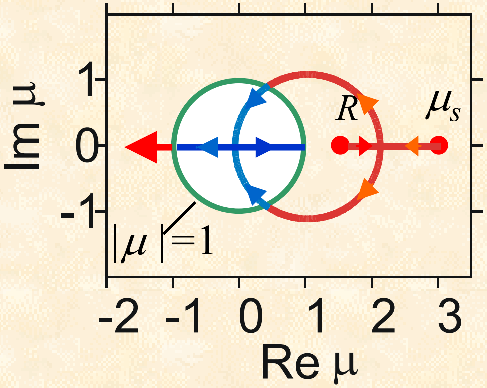
$$y_{n+1} = \mu_s y_n - KF_n, \quad \mu_s > 1$$

Extended controller:

$$F_n = y_n - y_{n-1} + R(y_{n-1} - y_{n-2}) + + R^2(y_{n-2} - y_{n-3}) + \ldots$$

$R < 1$ – **convergence condition**

Why do not try $R > 1$ **???**



It works !

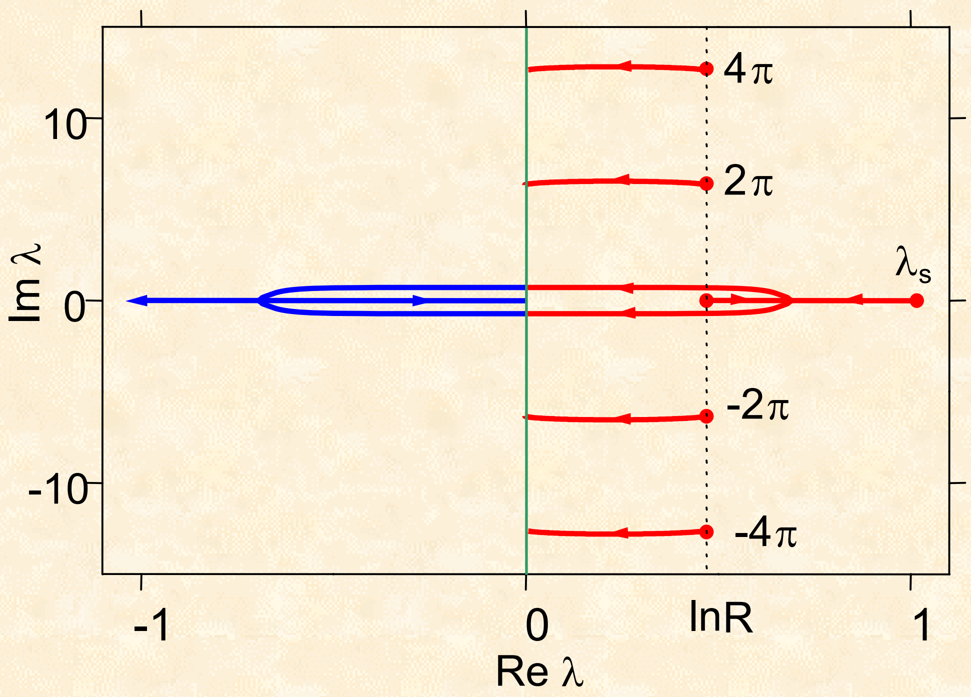
Extended Delayed Feedback Controller for R>1 (Simple Continuous Time System)

Unstable system:

$$dy/dt = \lambda_s y - KF(t)\,,\ \lambda_s > 0$$

Extended controller:

$$F(t) = y(t) - y(t-\tau) + R\,F(t-\tau)$$



$R > 1$

It does not work !

Unstable Extended Delayed Feedback Controller: UEDFC

(EDFC supplemented by an unstable degree of freedom)

Dynamic system:

$$\dot{\vec{x}} = \vec{f}(\vec{x}, \; p)$$

p *- control parameter*

$$y = g(\vec{x})$$

- output variable

UEDFC:

$$p = p_0 + K F_u(t)$$

$$F_u(t) = F(t) + w(t)$$

$$F(t) = y(t) - y(t-\tau) + R\,F(t-\tau)$$

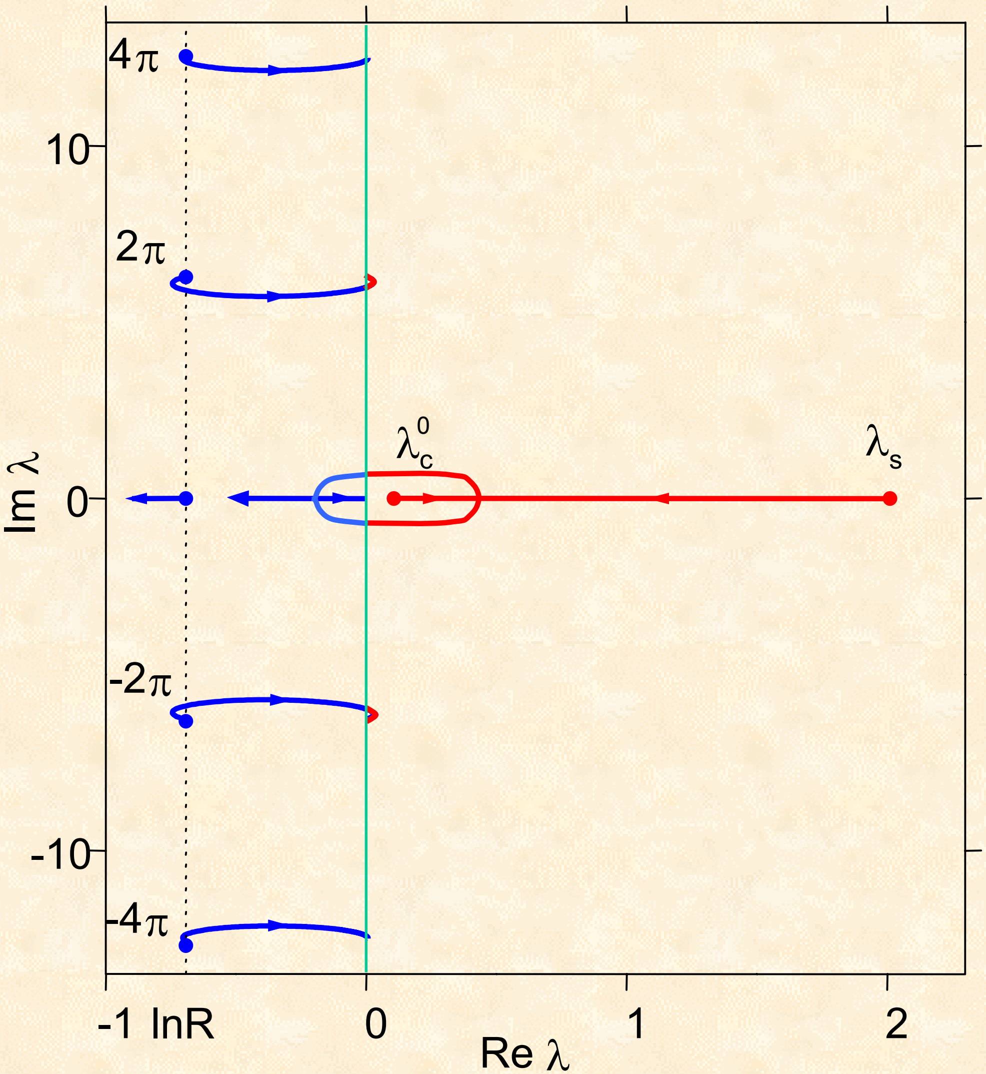
- usual EDFC $(R<1)$

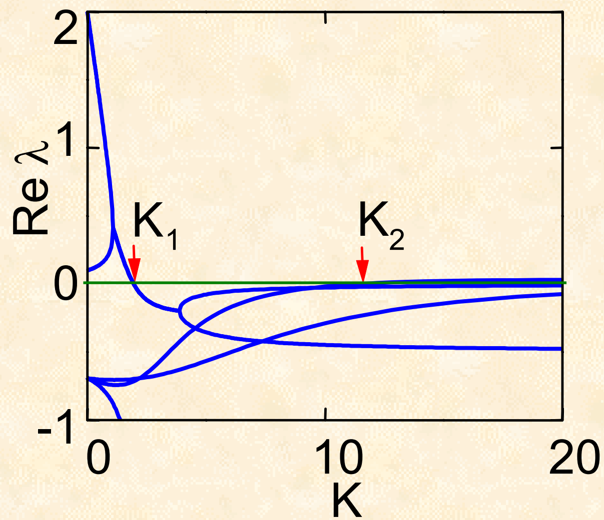
$$\dot{w} = \lambda_c^0 w(t) + \left(\lambda_c^0 - \lambda_c^\infty\right) F(t)$$

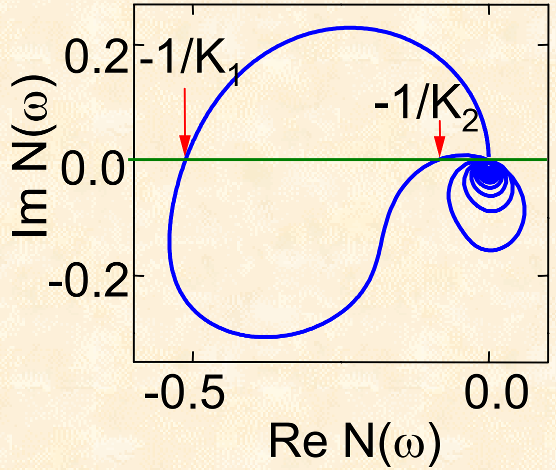
- additional unstable degree of freedom $(\lambda_c^0 > 0)$

Simple Continuous Time System under UEDFC

$$dy/dt = \lambda_s y - KF_u(t)\ ,\ \lambda_s > 0$$







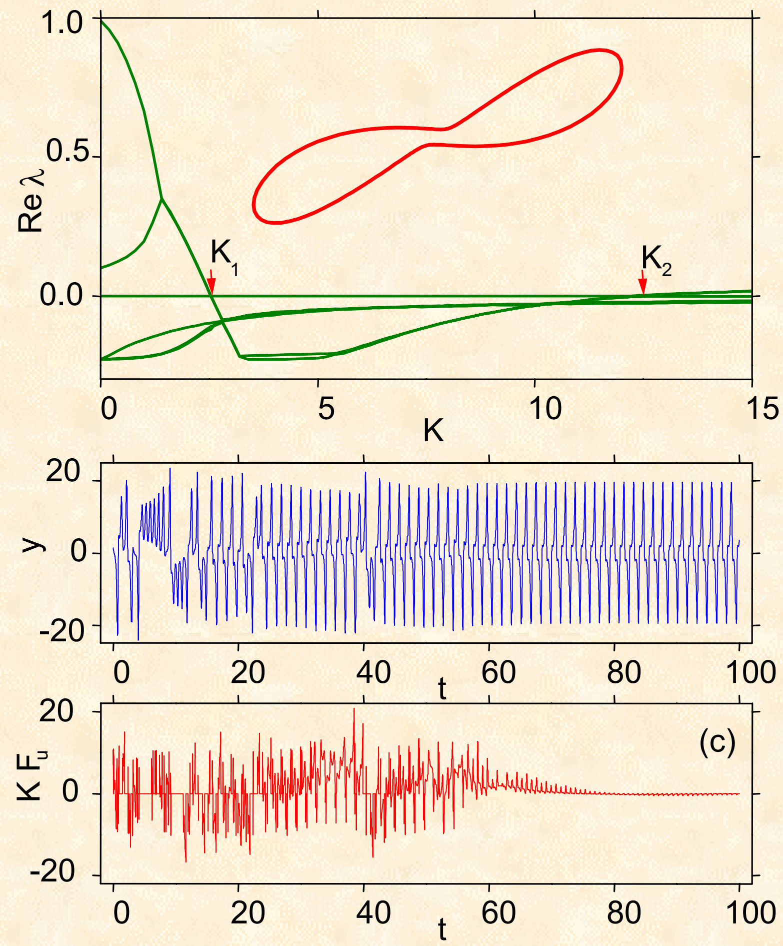
The Lorenz System under UEDFC

$$\dot{x} = -\sigma(y+z)$$
$$\dot{y} = rx - y - xz - KF_u(t)$$
$$\dot{z} = xy - bz$$

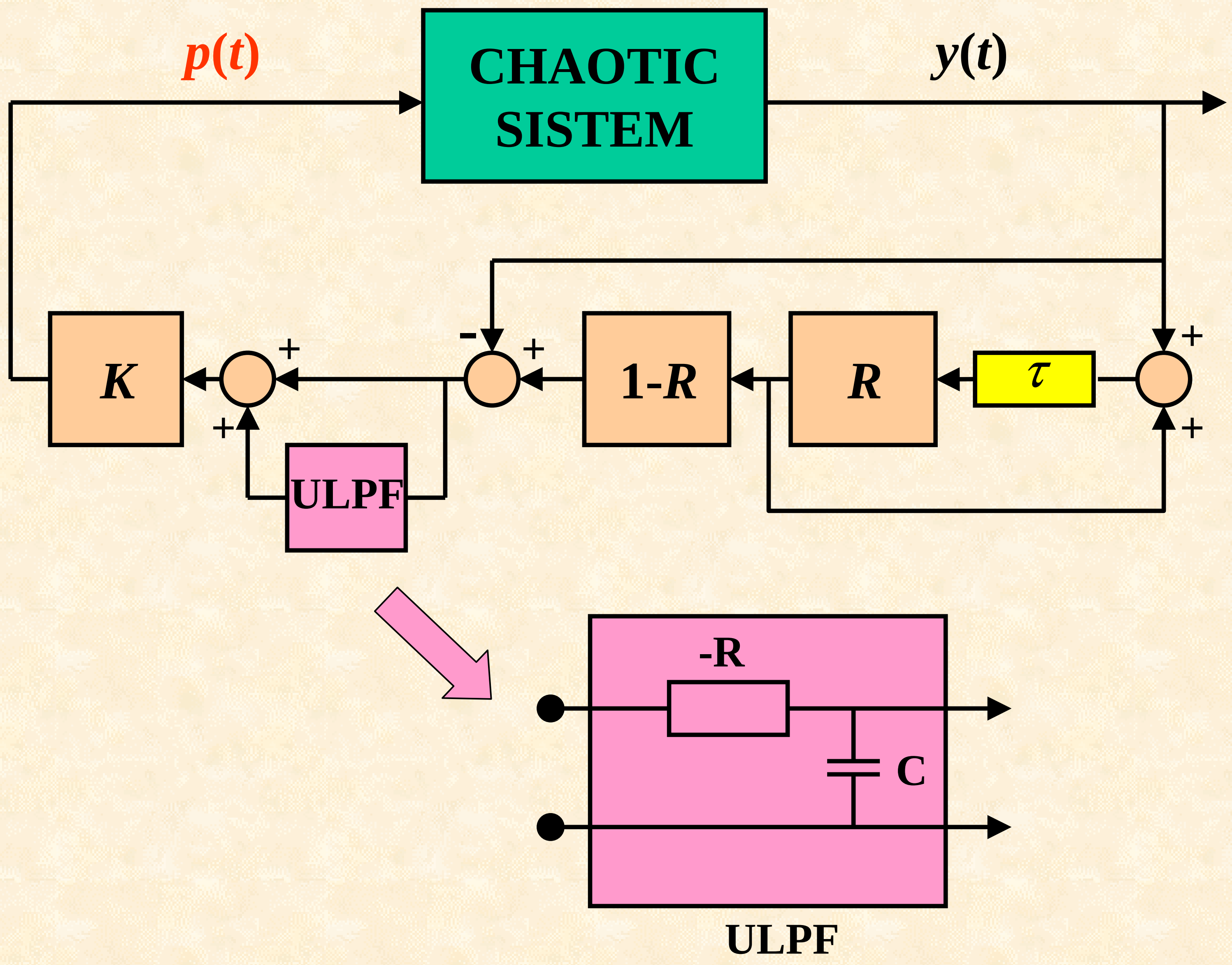
UEDFC:

$$F_u(t) = F(t) + w(t)$$
$$F(t) = y(t) - y(t-\tau) + R\,F(t-\tau)$$
$$\dot{w} = \lambda_c^0 w(t) + \left(\lambda_c^0 - \lambda_c^\infty\right) F(t)$$

$$R < 1, \quad \lambda_c^0 > 0$$



Block Diagram of the **UEDFC**



Stabilizing and tracking unknown steady states of dynamical systems

K.Pyragas, V.Pyragas, I.Kiss, J.Hudson, Phys.Rev.Lett., 244103 (2002)

Dynamic system:

$$\dot{\vec{x}} = \vec{f}(\vec{x}, p)$$

p *- control parameter*

$y = g(\vec{x})$ *- output variable*

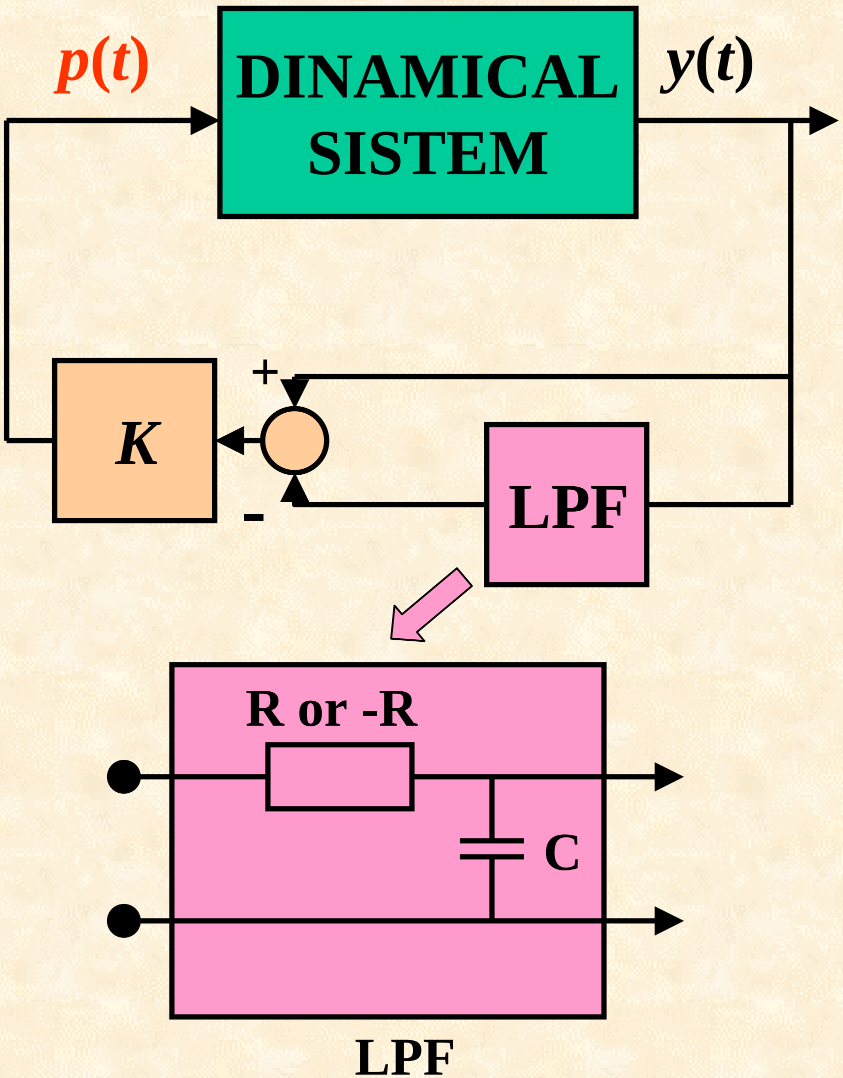
$$\vec{f}(\vec{x}_0, p_0) = 0$$

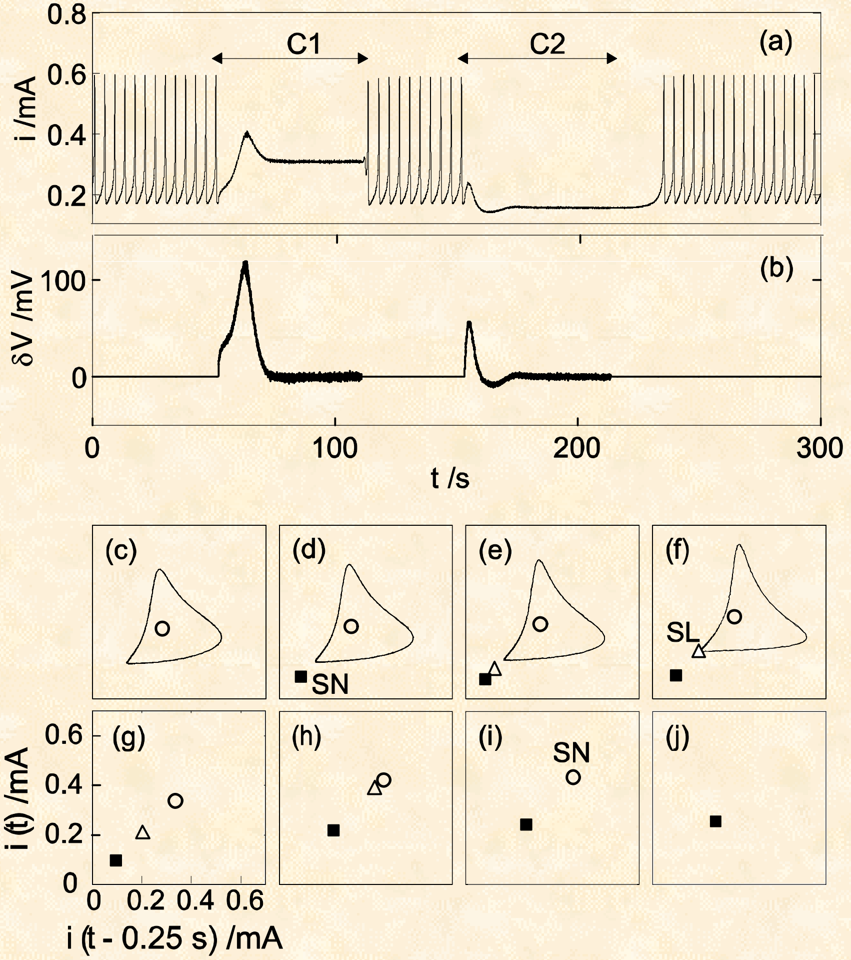
$\vec{x}_0$ *is a fixed point for* $p = p_0$

Adaptive controller: $p = p_0 + K(w-y)$ $dw/dt = \lambda(w-y)$

$\lambda > 0$ is a necessary condition to stabilize the steady states with an odd number of real positive eigenvalues!

Controlling a chemical reaction





Conclusions

- ***Time-delayed feedback control technique is a convenient and universal tool for various experimental applications.***

- ***An unstable degree of freedom introduced into a feedback loop can overcome the well known limitation of the delayed feedback schema. The modified controller can stabilize unstable periodic orbits with a finite torsion and can be successfully used for a wider class of chaotic systems.***

- ***The idea of using an unstable controller for stabilizing unstable states of dynamical systems seems to be new in control theory. It is surprising that the idea works since common sense dictates that the instability introduced into a feedback loop should increase the instability of the closed loop system. Nevertheless, two coupled unstable systems can stabilize each other and operate in a stable regime.***

Acknowledgments

Prof. Alexander Fradkov

for inviting me for this talk

My daughter Viktorija Pyragaitė

for preparing the demonstration program of two uncoupled double-pendulums