Phase lead synchronization in unidirectionally coupled chaotic systems

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FIZINIŲ IR TECHNOLOGIJOS MOKSLŲ CENTRAS

MOKSLAS • EKONOMIKA • SANGLAUDA EUROPOS SĄJUNGA
Kuriame Lietuvos ateitį

XXXI Dynamics Days Europe

Oldenburg, Germany September 12 - 16, 2011

Outline

- Introduction
- Overview of chaotic anticipating synchronization
- Phase lead synchronization
- From phase lead to anticipating synchronization
- Conclusions

Introduction (I)

Different types of chaotic synchronization:

- identical or complete (mutual coupling, identical systems)
- phase (mutual coupling, nonidentical systems)
- lag (unidirectional coupling, nonidentical systems)
- generalized (unidirectional coupling, nonidentical systems)
- anticipating (unidirectional coupling, identical systems)

Introduction (II)

The main aims of this talk are:

- to present an overview of anticipating synchronization
- to introduce the notion of phase lead synchronization in unidirectionally coupled chaotic systems without delay
- to demonstrate the transition:

Phase lead syncronization → Anticipating syncronization → Identical syncronization

which appears with the increase of the coupling strength

Overview of anticipating synchronization (I)

Idea of anticipating synchronization:

H. U. Voss, Phys. Rev. 61, 5115 (2000);
Phys. Rev.Lett. 87, 014102 (2001)

Two schemes for anticipating synchronization:

- the master system with a memory (contains time-delay terms)
- the master system without a memory

Overview of anticipating synchronization (II)

The master system with a memory:

$$\dot{x}_1 = -\alpha x_1 + f\left(x_1\left(t-\tau\right)\right)$$ – master (drive)

$$\dot{x}_2 = -\alpha x_2 + f\left(x_1\left(t\right)\right)$$ – slave (response)

$x_1(t) = x_2(t-\tau)$ *or* $x_2(t) = x_1(t+\tau)$ – anticipatory manifold

$\Delta(t) = x_1(t) - x_2(t-\tau)$ – transversal deviation from anticipatory manifold

$\dot{\Delta} = -\alpha \Delta$ $\Delta \xrightarrow[t\to\infty]{} 0$ for $\alpha > 0$

$x_2(t) \xrightarrow[t\to\infty]{} x_1(t+\tau)$ – the slave anticipates by amount τ the output of the master

Anticipation time = Memory time of the master system

Overview of anticipating synchronization (III)

The master system without a memory:

$$\dot{\vec{r}}_1 = \vec{f}(\vec{r}_1)$$ – master (drive)

$$\dot{\vec{r}}_2 = \vec{f}(\vec{r}_2) + K[\vec{r}_1(t) - \vec{r}_2(t-\tau)]$$ – slave (response)

$\vec{r}_1(t) = \vec{r}_2(t-\tau)$ *or* $\vec{r}_2(t) = \vec{r}_1(t+\tau)$ – anticipatory manifold

$\vec{\Delta}(t) = \vec{r}_1(t) - \vec{r}_2(t-\tau)$ – transversal deviation from anticipatory manifold

$$\dot{\vec{\Delta}} = \left.\frac{\partial \vec{f}}{\partial \vec{x}_2}\right|_{\vec{x}_2 = \vec{x}_1(t+\tau)} \vec{\Delta} - K\vec{\Delta}(t-\tau)$$

– equation for transversal Lyapunov exponents (TLE) (necessary stability condition: $\lambda_\perp < 0$; $\lambda_\perp$ – maximum TLE)

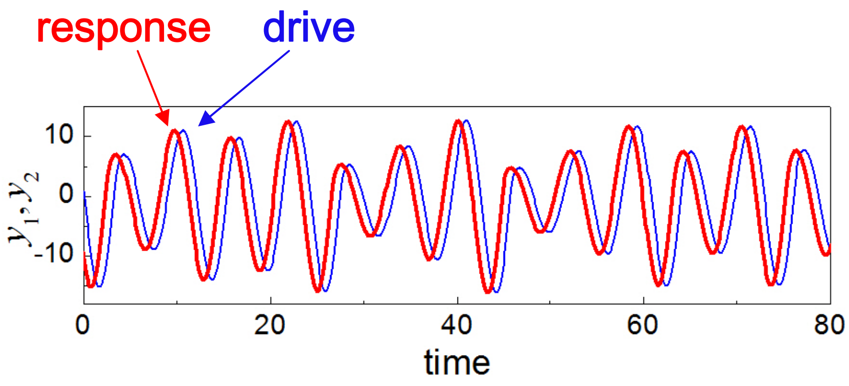
Anticipating synchronization without a memory in the master system is the most interesting phenomenon for various practical applications, especially for a real-time forecasting of a given chaotic dynamics

Limitation of the delay coupling scheme

Usually a diagonal coupling is used: $K = diag(1, \cdots, 1)$

In this case the long-term prediction is impossible

Example: Rössler systems



The maximum stably anticipation time is much shorter than the characteristic time scales of the system's dynamics.

Overview of anticipating synchronization (V)

Prolonging the prediction via a chain of slave systems

(Voss, Phys. Rev.Lett. 87, 014102 (2001))

$$
\begin{aligned}
\dot{\vec{r}}_1 &= \vec{f}(\vec{r}_1) \\
\dot{\vec{r}}_2 &= \vec{f}(\vec{r}_2) + K\left[\vec{r}_1 - \vec{r}_2(t-\tau)\right] \\
\dot{\vec{r}}_3 &= \vec{f}(\vec{r}_3) + K\left[\vec{r}_2 - \vec{r}_3(t-\tau)\right] \\
&\vdots \\
\dot{\vec{r}}_{N+1} &= \vec{f}(\vec{r}_{N+1}) + K\left[\vec{r}_N - \vec{r}_{N+1}(t-\tau)\right]
\end{aligned}
$$

$\Rightarrow$ $\tau_N = N\tau$

- C. Mendoza, S. Boccaletti, and A. Politi, Phys. Rev. E **69**, 047202 (2004)

The chain is unstable to propagating perturbations (**convectivelike instability**). This limits the number of slaves in the chain which can operate in a stable regime.

Phase-lag compensating coupling (PLCC)

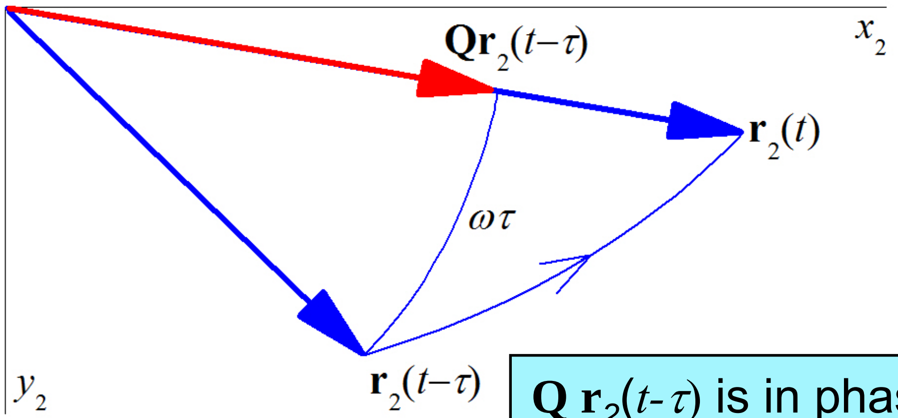
(K. Pyragas, and T. Pyragienė, Phys. Rev. E **78**, 046217 (2008))

$$\dot{\vec{r}}_1 = \vec{f}(\vec{r}_1)$$
$$\dot{\vec{r}}_2 = \vec{f}(\vec{r}_2) + K[\vec{r}_1(t) - \vec{r}_2(t-\tau)]$$

Stabilizing term

$$K = kQ$$

k – (scalar) feedback strength



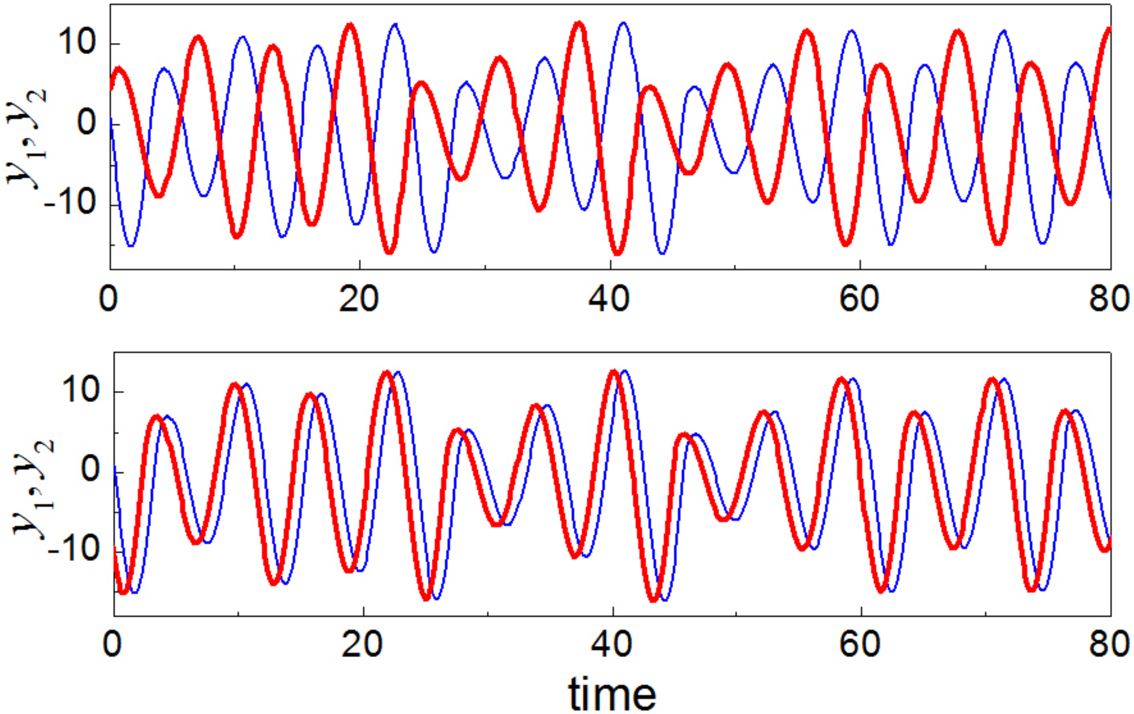
$\mathbf{Q}\ \mathbf{r}_2(t\text{-}\tau)$ is in phase with $\mathbf{r}_2(t)$!!!

$$Q = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

– projects the vector field on the unstable (x,y) plane and rotates this projection by the angle α

$\alpha = \omega\tau$, ω – frequency of the unstable spiral

PLCC vs. DC (Rössler systems)



⇦ PLCC (max. prediction)

$$\tau_{PLCC} \approx 4\tau_{DC}$$

⇦ Diagonal coupling (DC) (max. prediction)

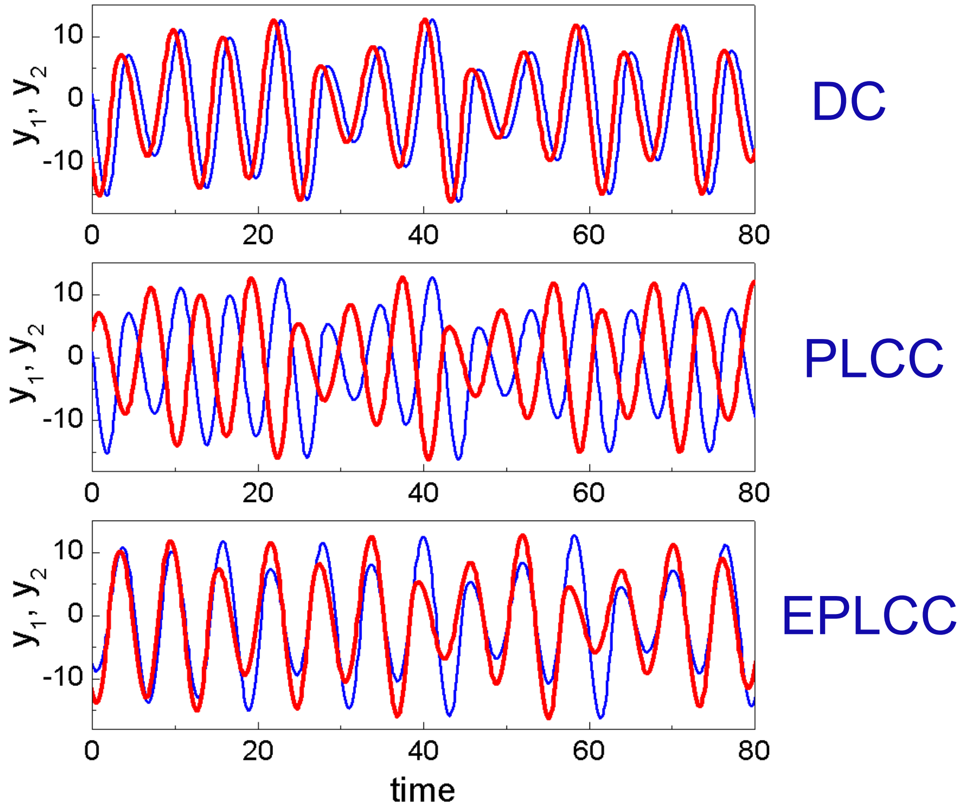
PLCC allows us to make prediction (with the single slave system) for more than half of characteristic period of chaotic oscillations.

Overview of anticipating synchronization (VIII)

Extended phase-lag compensating coupling (EPLCC)

(K. Pyragas, T. Pyragienė, Phil. Trans. R. Soc. A 368, 305 (2010))

EPLCC incorporates in the feedback loop many previous states of the system similar to the extended delayed feedback control algorithm (EDFC)



The maximum prediction time attained with EPLCC is equal to the characteristic period of chaotic oscillations

How general is the phenomenon of anticipating synchronization?

It seems that anticipating synchronization may appear only in man-made systems, since there are many artificial requirements:

- the drive and response systems have to be identical
- the coupling has to be designed in a specific form
- the specific memory (delay) elements have to be introduced either in the drive or response system

Can anticipating synchronization appear in natural (not man-made) systems, say in neural systems ?

By analyzing the phase synchronization in unidirectionally coupled non-identical chaotic systems without delays we have detected the phase lead synchronization, which leads to anticipating synchronization

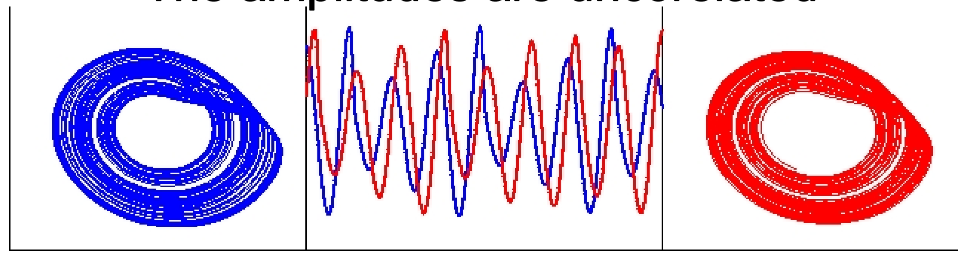
Phase synchronization

M. Rosenblum, A. Pikovsky, J. Kurths, PRL **76**, 1804 (1996)

Mutual coupling

ω_1 Chaotic system ⇄ ω_2 Chaotic system

Phases are locked
The amplitudes are uncorelated



Is it possible to obtain a phase lead synchronization in unidirectionally coupled non-identical chaotic systems?

uniderectional coupling

ω_1 Chaotic drive → ω_2 Chaotic response

Can the phases be locked such that

$$\varphi_2 > \varphi_1 \,?$$

Phase lead synchronization (PLS) in regular oscillators

$$\dot{\varphi}_1 = \omega_1,$$
$$\dot{\varphi}_2 = \omega_2 + \varepsilon \sin(\varphi_1 - \varphi_2)$$

$$\omega_2 > \omega_1$$

$$\theta = \varphi_2 - \varphi_1$$

$$\dot{\theta} = \omega_2 - \omega_1 - \varepsilon \sin\theta$$ ⇦ Adler equation

$$\dot{\theta} = 0 \quad \Rightarrow \quad \varphi_2 - \varphi_1 = \arcsin((\omega_2 - \omega_1)/\varepsilon) > 0$$

If the response system is faster than the drive system than their phases are locked in such a way that the phase of the response system is ahead of the drive phase

PLS in unidirectionally coupled chaotic Rössler systems

$\omega_2 > \omega_1$

drive

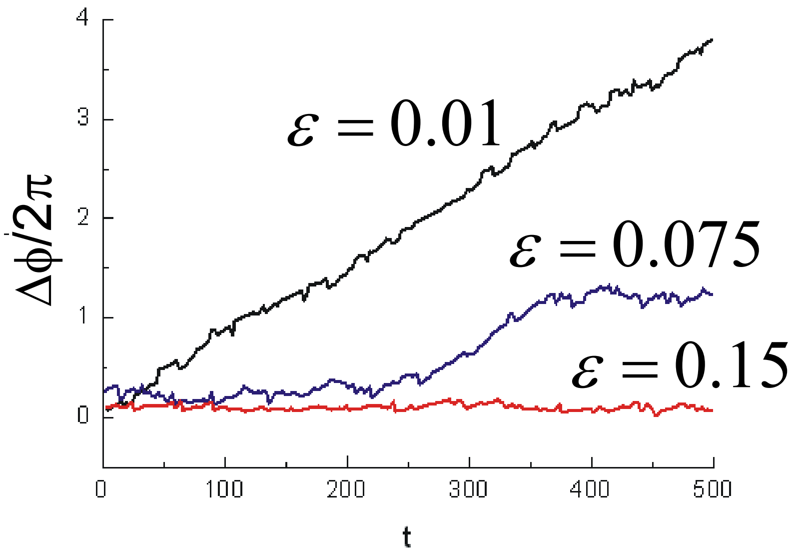
$$\dot{x}_1 = -\omega_1 y_1 - z_1,$$
$$\dot{y}_1 = \omega_1 x_1 + a y_1,$$
$$\dot{z}_1 = b + z_1(x_1 - c).$$

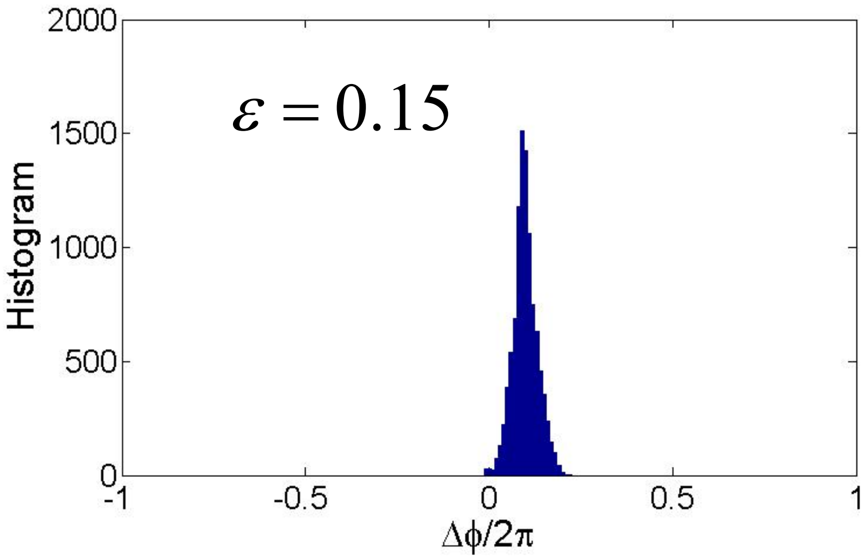
response

$$\dot{x}_2 = -\omega_2 y_2 - z_2 + \varepsilon(x_1 - x_2),$$
$$\dot{y}_2 = \omega_2 x_2 + a y_2,$$
$$\dot{z}_2 = b + z_2(x_2 - c).$$

$$\varphi_{1,2} = \arctan(y_{1,2} / x_{1,2})$$

$$\Delta\varphi = \varphi_2 - \varphi_1$$

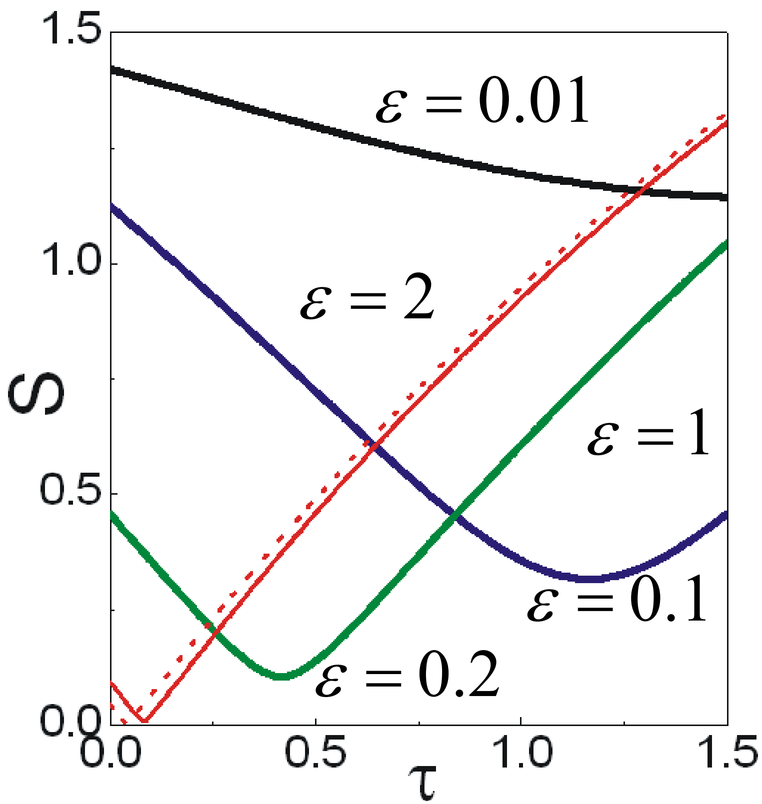




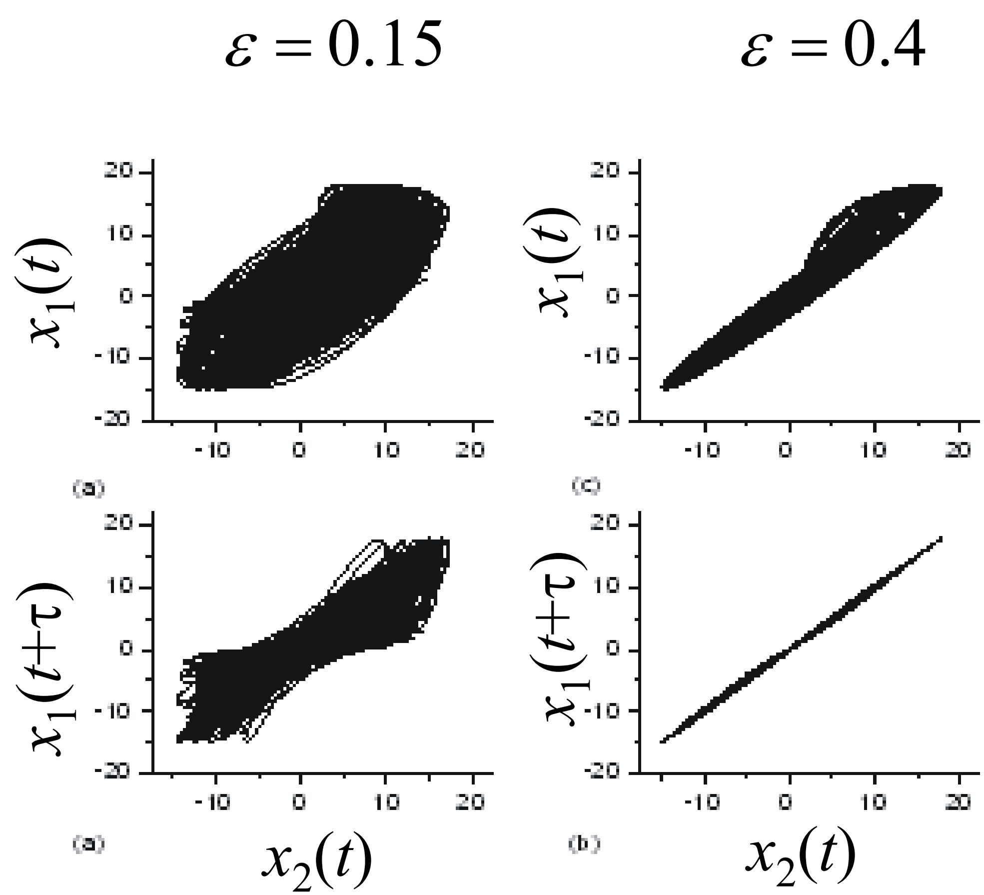
From phase lead to anticipating synchronization

Similarity function

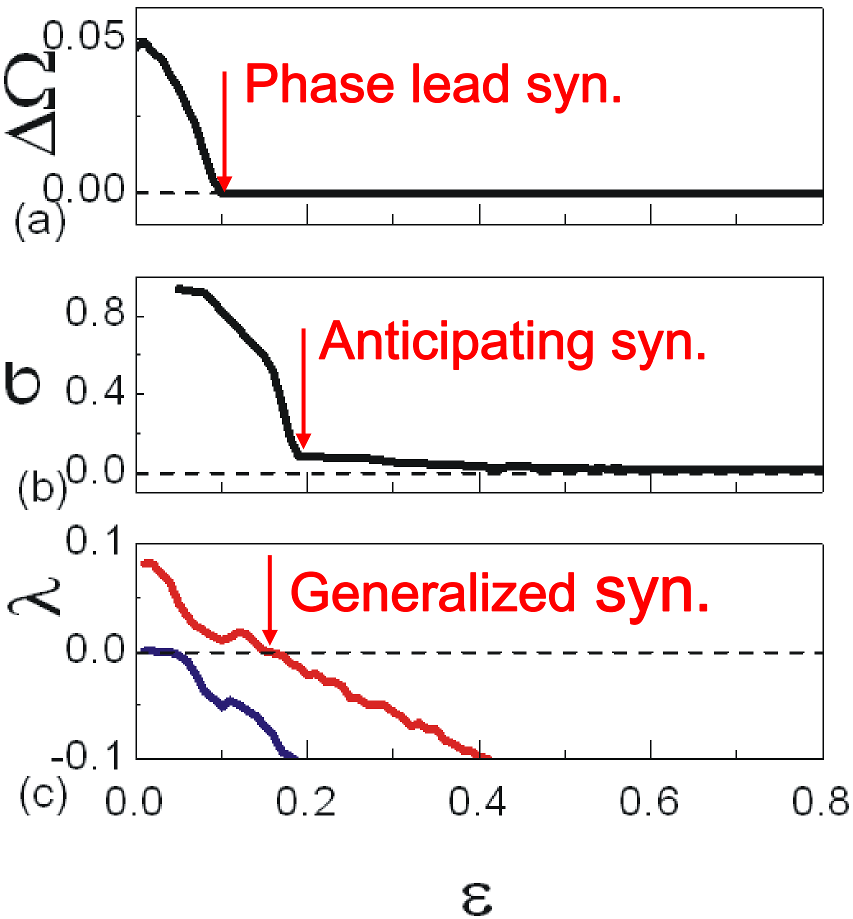
$$S = \frac{<[x_2(t) - x_1(t+\tau)]^2>}{[<x_1{}^2><x_2{}^2>]^{1/2}}$$



Phase portraits



Transitions with the increase of the coupling strength (I)

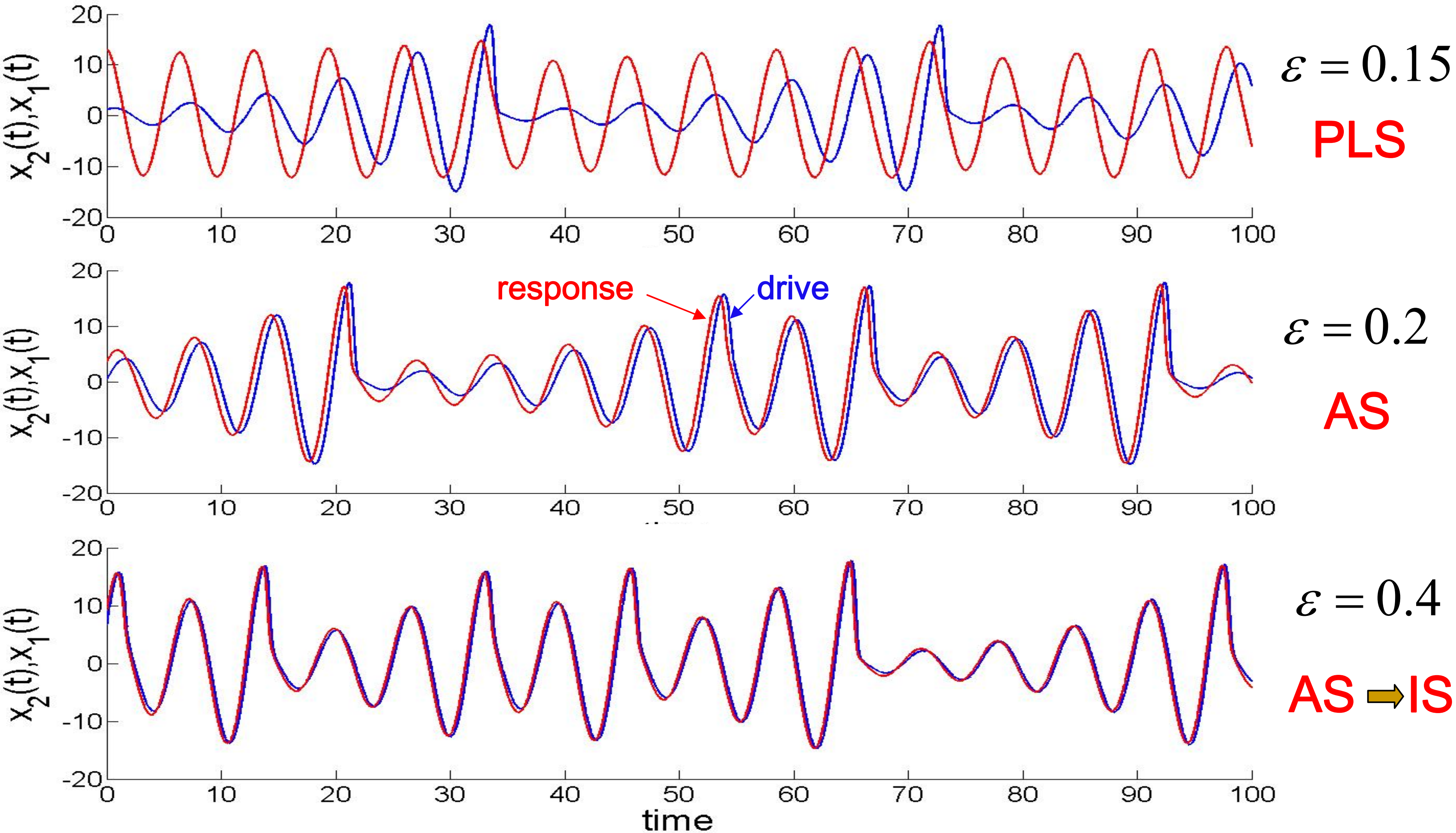


$$\Omega_i = \langle \dot{\varphi}_i \rangle, \Delta\Omega = \Omega_2 - \Omega_1$$

$$\sigma = \min S(\tau)$$

Two largest conditional Lyapunov exponents

Transitions with the increase of the coupling strength (II)



Anticipating synchronization of chaotic nonidentical Hindmarsh-Rose neurons (I)

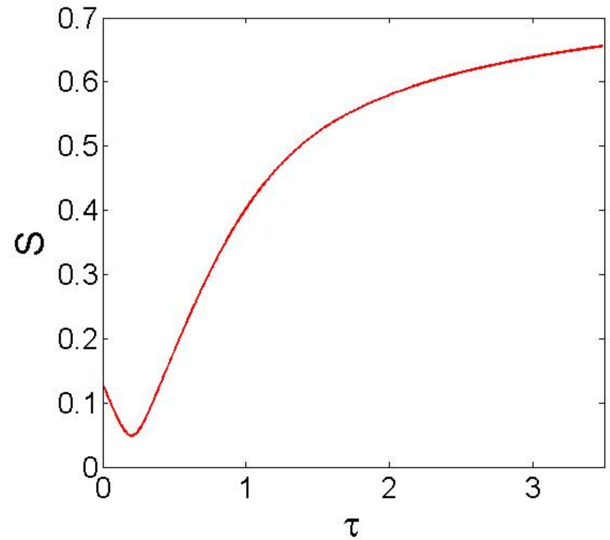
drive

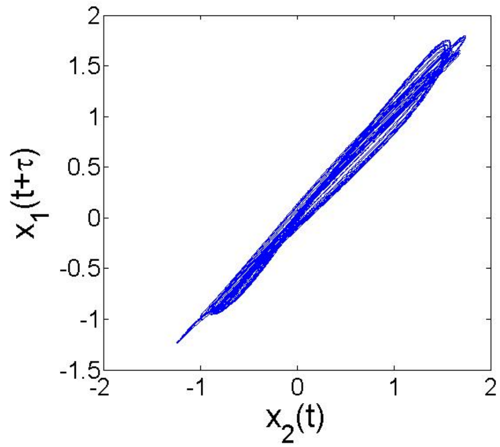
$$C_1\dot{x}_1 = y_1 + x_1^2(b - ax_1) - z_1 + J$$
$$\dot{y}_1 = c - dx_1^2 - y_1$$
$$\dot{z}_1 = r(s(x_1 - x_{st})) - z_1$$

$$C_2 < C_1$$

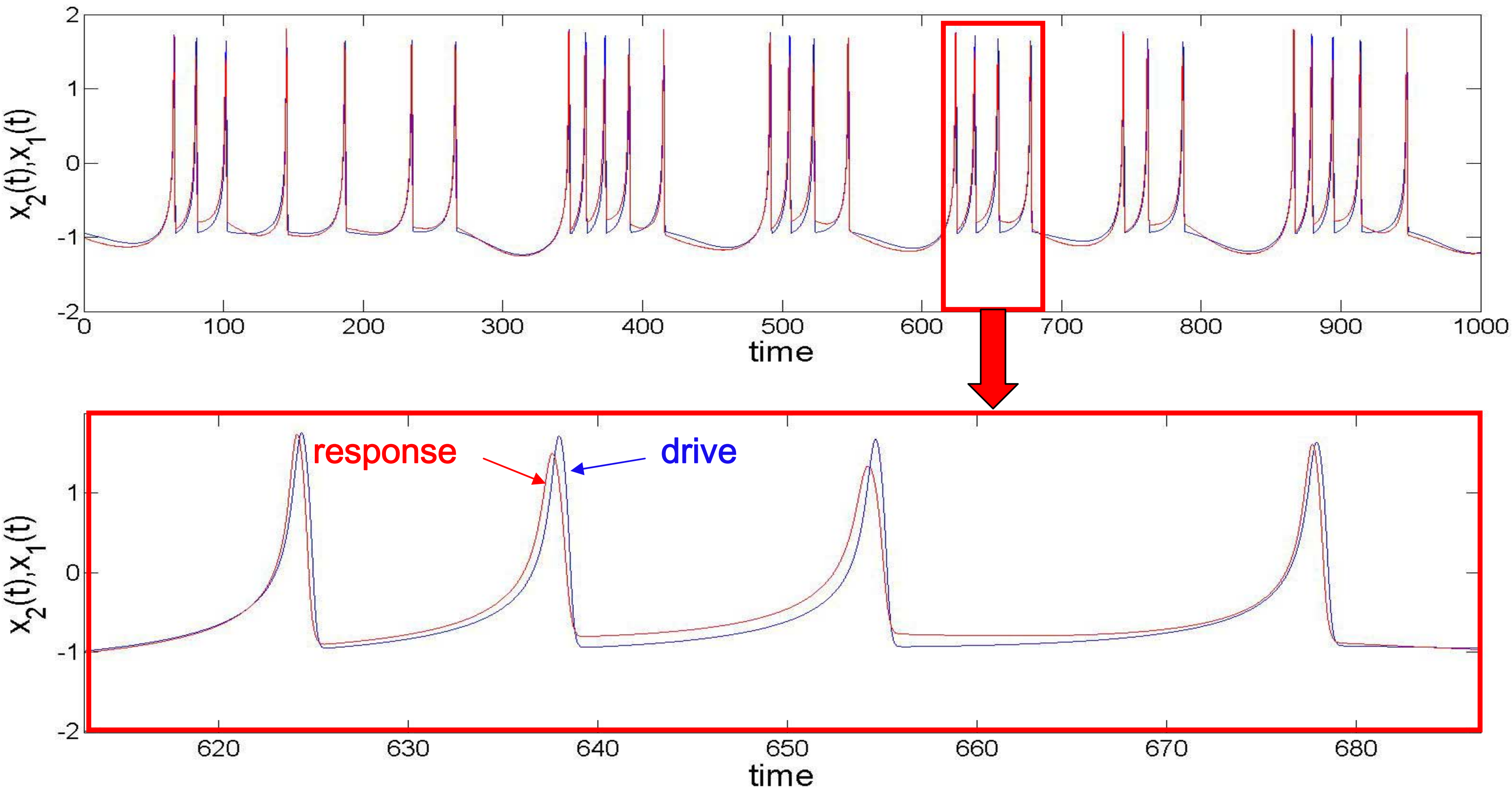
response

$$C_2\dot{x}_2 = y_2 + x_2^2(b - ax_2) - z_2 + J + \varepsilon(x_1 - x_2)$$
$$\dot{y}_2 = c - dx_2^2 - y_2$$
$$\dot{z}_2 = r(s(x_2 - x_{st})) - z_2$$





Anticipating synchronization of chaotic nonidentical Hindmarsh-Rose neurons (II)



Conclusions

- **The effect of phase lead synchronization may appear in non-indeintical unidirecionally coupled chaotic systems provided the response system is faster than the drive system**

- For sufficiently strong coupling strength, the phase lead synchronization transforms to the anticipating synchronization. As a result, the anticipating synchronization may take place in non-identical systems without delays

- The anticipation mechanism based on the phase lead synchronization maybe is explored in natural neural networks to compensate the communication delays between neurons, which appear due to finite speed propagation of information in axons.

Acknowledgments

Organizers of the “XXXI Dynamic Days Europe” conference for financial support

Organizers of the mini-symposium “Collective dynamics of coupled systems” for invitation

The support from the global grant Nr. VP1-3.1-ŠMM-07-K-01-025