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The anticipating of chaotic dynamics via an act-and-wait control

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FIZINIŲ IR TECHNOLOGIJOS MOKSLŲ CENTRAS
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Kuriame Lietuvos ateitį

Outline

- Background
- Simple model
- Chaotic systems
- Conclusions

Anticipating synchronization

Drive system

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right)$$

$\mathbf{f}(\mathbf{r}(t))$ - a nonlinear vector function

$\mathbf{r}_{1,2} = (x_{1,2}, y_{1,2}, z_{1,2})$ - state vectors

Response system

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) - \mathbf{K}[\mathbf{r}_2(t-\tau) - \mathbf{r}_1(t)]$$

K=k diag(1,1,1**)**

k - coupling strength

The present state of the response system $\mathbf{r}_2(t)$

coincides with the future state of the drive $\mathbf{r}_1(t+\tau)$

↓

The response system anticipates the dynamics of the drive

H. Voss Phys. Rev. E **61**, 2000

Drive system

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right)$$

Response system

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) - \mathbf{K}[\mathbf{r}_2(t-\tau) - \mathbf{r}_1(t)]$$

The anticipating manifold $\mathbf{r}_2(t) = \mathbf{r}_1(t+\tau)$

is stable only for some anticipating time and coupling strength k

Time-delay τ = anticipation time

The presence of the time-delay term complicates the stability analysis

H. Voss Phys. Rev. E **61**, 2000

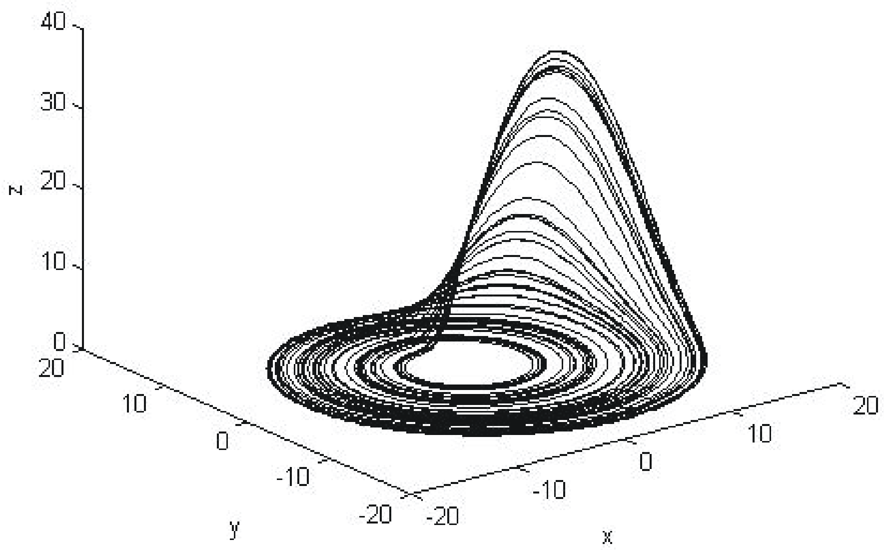
Problem

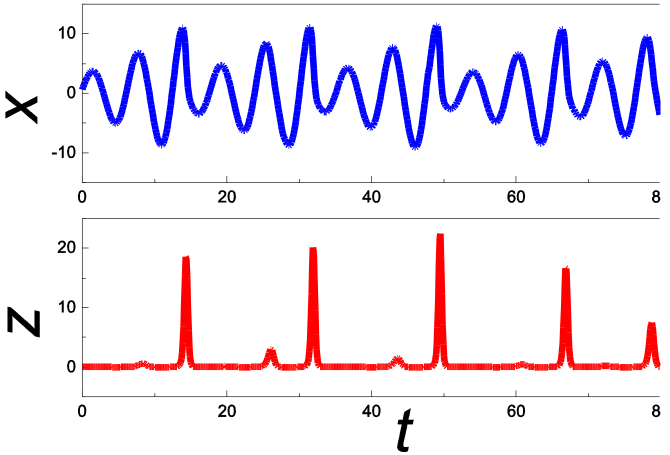
Due **to the time-delay term**, the anticipating manifold is defined by the **infinite** number of Lyapunov exponents

Solution

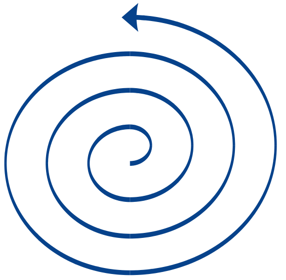
Transformation of the infinite system's phase space to the finite one via applying an **act-and-wait control**

The Rössler system





The system spends the most time in (x,y) plane.
Spiral – simple model



The Rössler system

$$\mathbf{f}(\mathbf{r}) = [-y - z, x + ay, b + z(x - c)], \mathbf{r} = (x, y, z)$$

$$a = 0.15, b = 0.2, c = 10$$

$$\downarrow \quad z = 0$$

Spiral: $\mathbf{f}(\mathbf{r}) = [-y, x + ay], \mathbf{r} = (x, y)$

$$x(t) \sim \mathrm{e}^{\gamma t} \cos(\omega t)$$

$$\gamma = \frac{a}{2} = 0.075, \omega = \sqrt{1 - \gamma^2} = 0.997$$

Act-and-wait control concept

Drive system

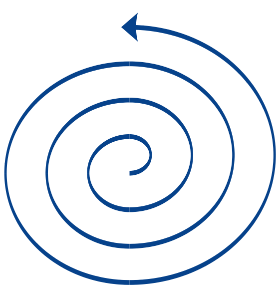
$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right)$$

Response system

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + u(t)\mathbf{I}$$

$$\boxed{u(t) - ?} \quad \mathbf{I} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Spiral system



$$\mathbf{f}(\mathbf{r}) = [-y, x + ay], \mathbf{r} = (x, y)$$

Scalar coupling

$$u(t) = -kG(t)S$$

$$S = \sum_{i=0,1} [a_i x_2(t - \tau - \theta_i) - x_1(t - \theta_i)]$$

k-coupling strength

$$\theta_i < \tau$$

τ - time-delay

θ_i - additional time-delay

$$\theta_0 = 0,\ a_0 = 1,$$

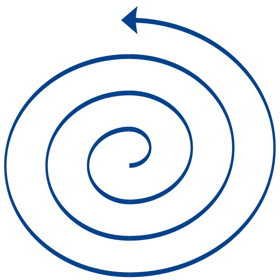
$$\theta_1 = \theta,\ a_1 - ?$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right) \qquad \dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + u(t)\mathbf{I}$$

Spiral approximate coupling

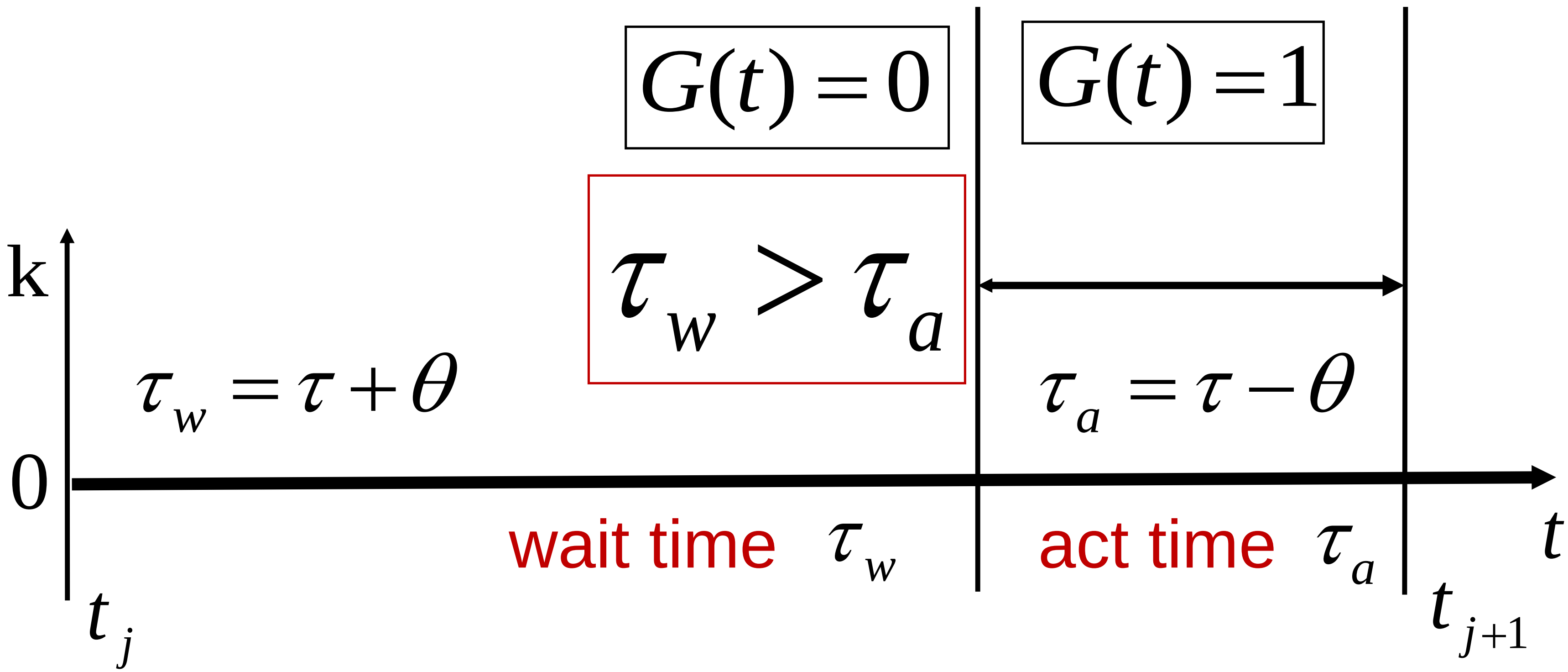
$$x^{s}(t) = x_0 + Ae^{\gamma t}\cos(\omega t + \varphi_0)$$

$$\downarrow$$



$$a_1{}^{s} = \frac{e^{\gamma \tau_w}\sin(\omega \tau)}{e^{\gamma \tau}\sin(\omega \tau_w)} \qquad \tau_w = \tau + \theta$$

The feedback loop is switched off and on periodically



$$T = t_{j+1} - t_j = \tau_w + \tau_a = 2\tau$$

T. Insperger (2006); G. Stépán., T. Insperger (2006)

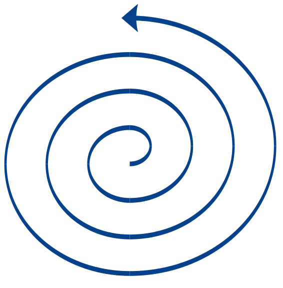
$$\dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + u(t)\mathbf{I} \qquad u(t) = -kG(t)S$$

$$S = \sum_{i=0,1} [a_i x_2(t-\tau-\theta_i) - x_1(t-\theta_i)]$$

In the acting period, the variables $x(t-\tau-\theta), x(t-\theta)$ are known variables of the waiting period

The coupling term $u(t)$ is a function of the time and does not depend on the delay

Spiral



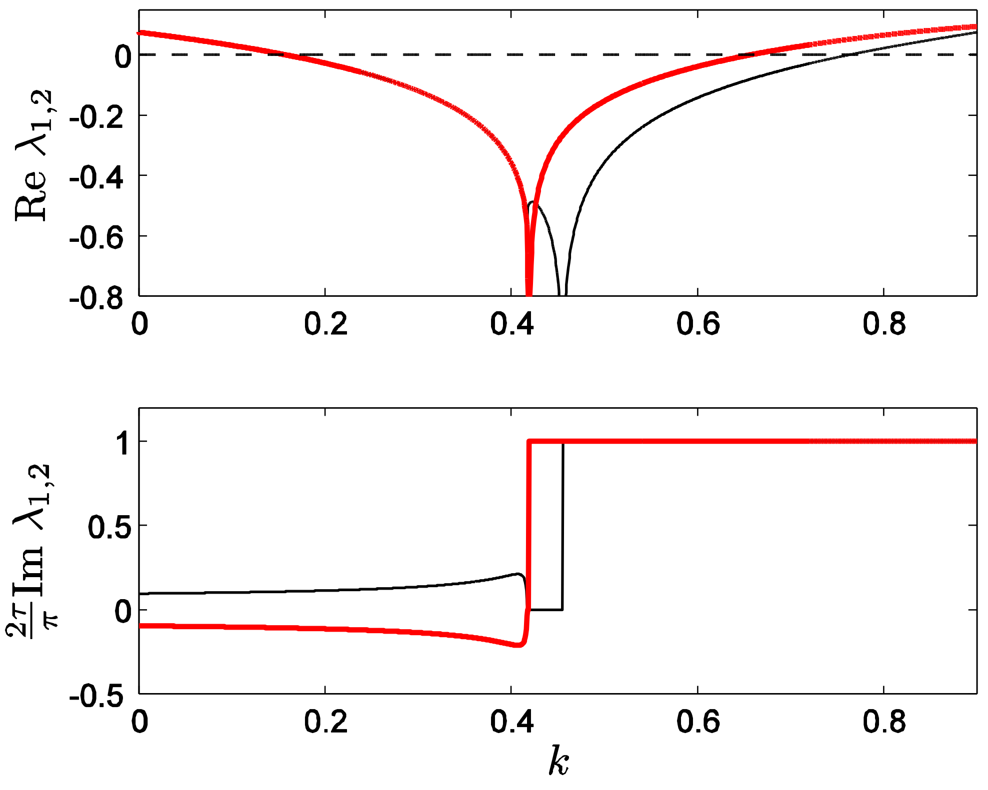
Variation equations for a deviation

$$\delta\mathbf{r}(t) = \mathbf{r}_2(t) - \mathbf{r}_1(t+\tau)$$

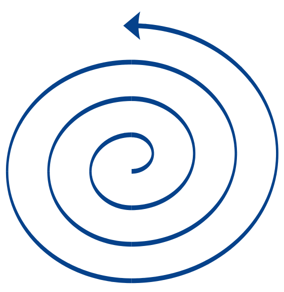
$$\boxed{\delta\mathbf{r}_{j+1} = \mathbf{M}\delta\mathbf{r}_j}$$

The anticipating manifold is defined by **2** Floquet exponents of the 2x2 monodromy matrix **M**

Floquet exponents



$\tau = 3,\ \theta = 0.1$

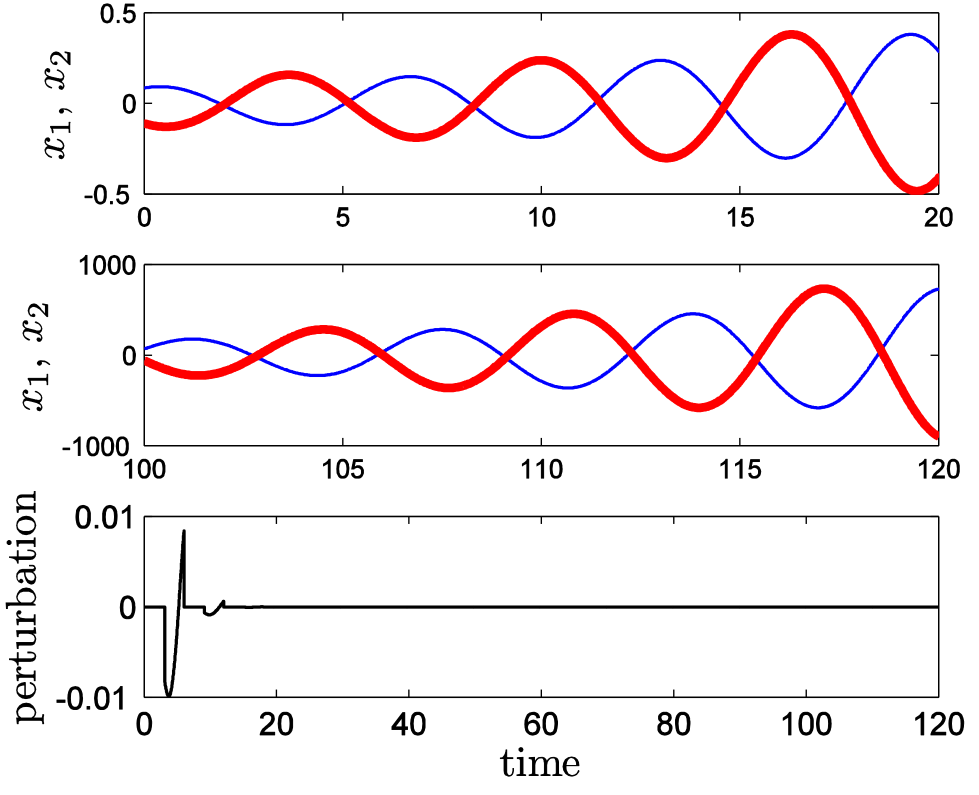


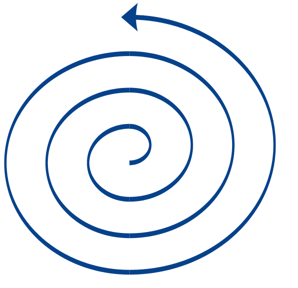
Optimal control strength $k_0 = 0.42$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\big(\mathbf{r}_1(t)\big) \qquad \dot{\mathbf{r}}_2(t) = \mathbf{f}\big(\mathbf{r}_2(t)\big) + u(t)\mathbf{I}$$

Anticipation

$$\tau = 3,\ \theta = 0.1 \qquad k_0 = 0.42$$





$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\big(\mathbf{r}_1(t)\big) \qquad \dot{\mathbf{r}}_2(t) = \mathbf{f}\big(\mathbf{r}_2(t)\big) + u(t)\mathbf{I}$$

Anticipating of chaotic dynamics

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right) \qquad \dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + u(t)\mathbf{I} \qquad G(t) = 0{,}1$$

$$u(t) = -kG(t)\{x_2(t-\tau) - x_1(t) + a_1[x_2(t-\tau-\theta) - x_1(t-\theta)]\}$$

We have a simple ODE system for the deviation: $\delta\dot{\mathbf{r}}_1(t) = \mathbf{J}(t)\delta\mathbf{r}_1(t)$

The solution of this system is $\delta\mathbf{r}(t) = \mathbf{\Phi}(t)\delta\mathbf{r}_0,\ \delta\mathbf{r}_0 = \delta\mathbf{r}(0)$

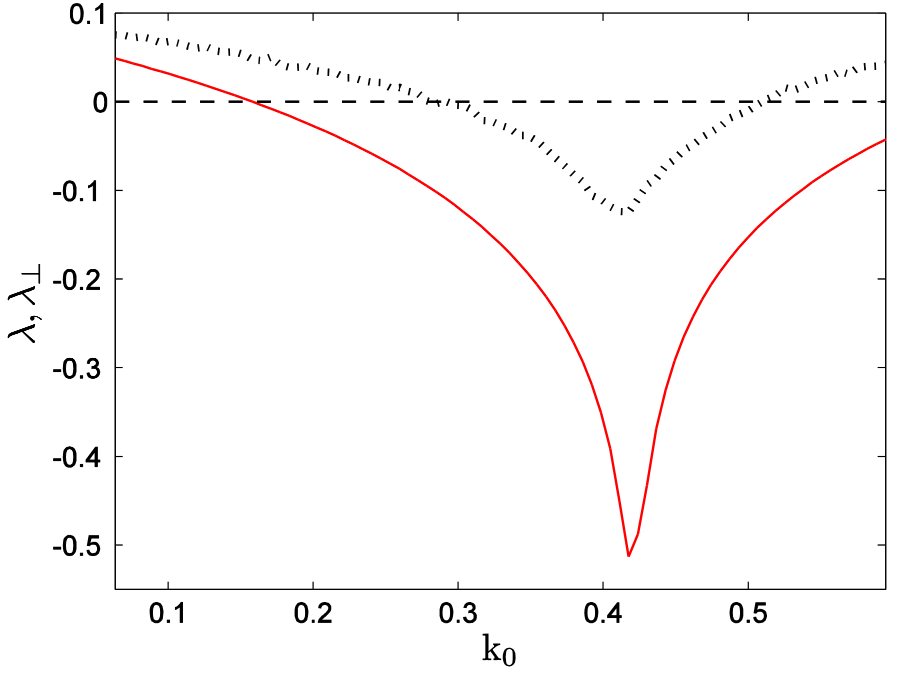
$\mathbf{\Phi}(t)$ - the evolution operator, **dxd** matrix.
It satisfys the same ODE, with $\mathbf{\Phi}(0) = \mathbf{I}$

Maximal transversal Lyapunov exponent $\lambda = \lim \frac{1}{2\tau j} \log\left(\frac{\| \delta r_j \|}{\| \delta r_0 \|}\right),\ j \to \infty$

The anticipating manifold is defined by the finite number of Lyapunov exponents!

The chaotic Rössler and simple spiral systems

Maximum transversal Lyapunov exponents for the spiral systems (red line) and the Rössler systems (black dots)



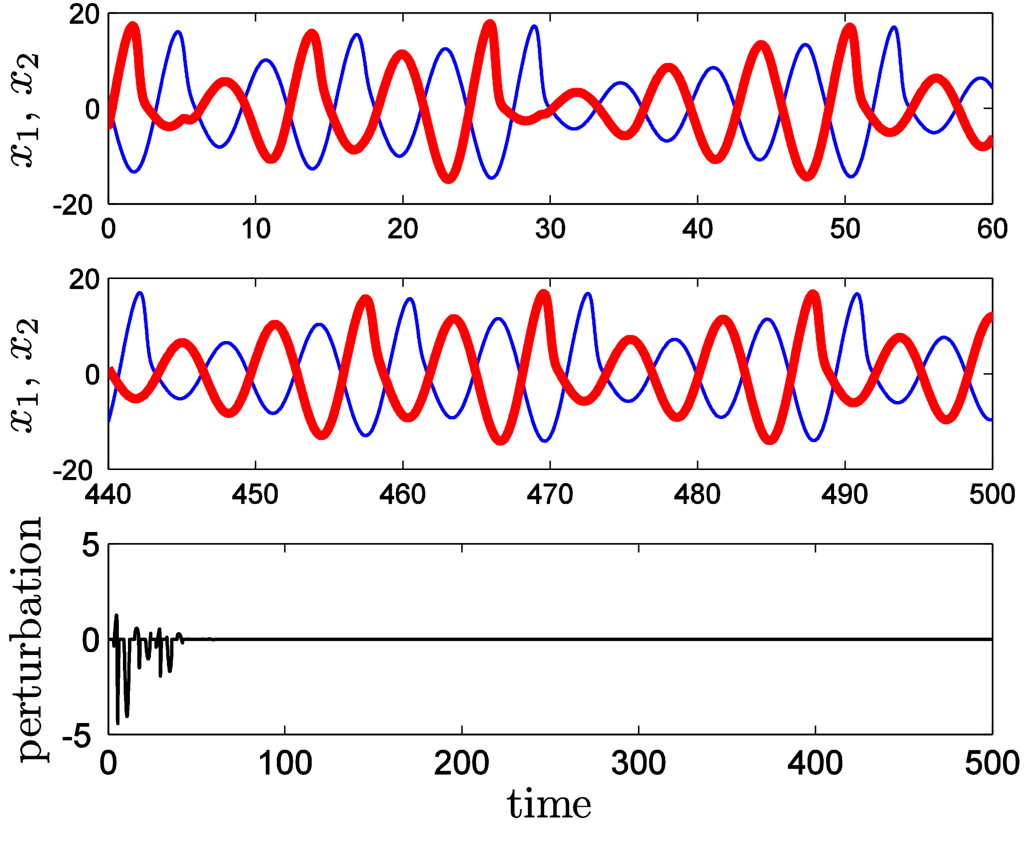
$\tau = 3, \ \theta = 0.1$

Optimal control strength

$$k_0 = 0.42$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right) \quad \dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + u(t)\mathbf{I}$$

The Rössler systems



$\tau = 3,\ \theta = 0.1$

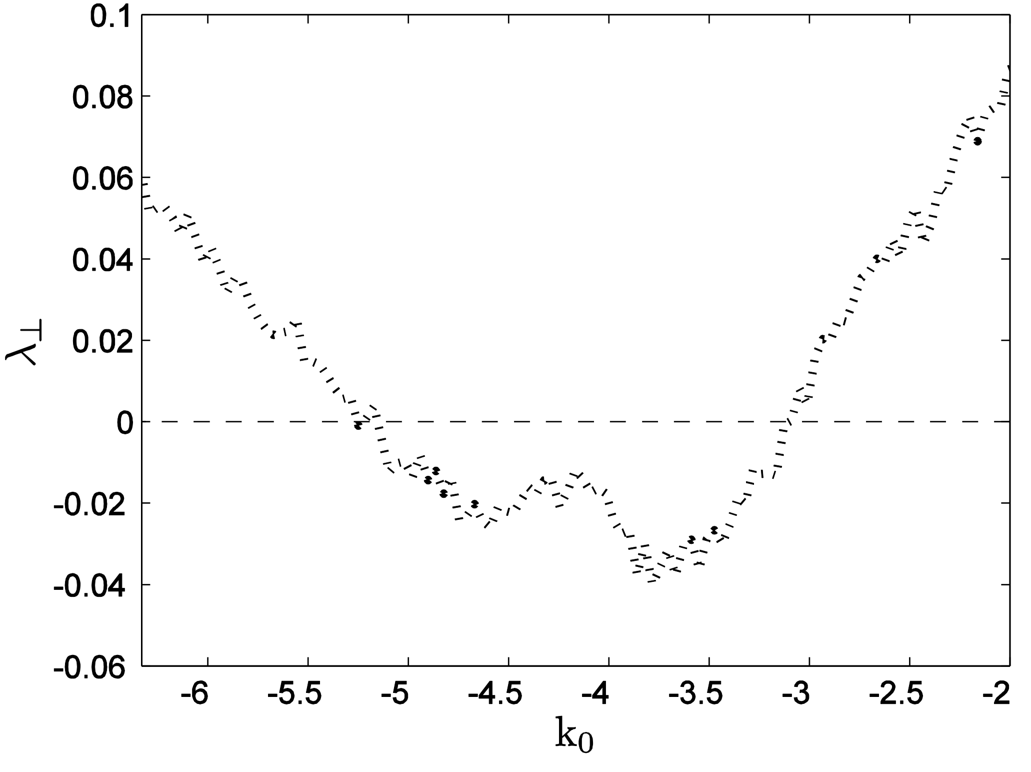
$k_0 = 0.42$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right) \quad \dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + u(t)\mathbf{I}$$

The Chua systems

Maximum transversal Lyapunov exponent

$$\tau = 2.5,\ \theta = 0.2$$

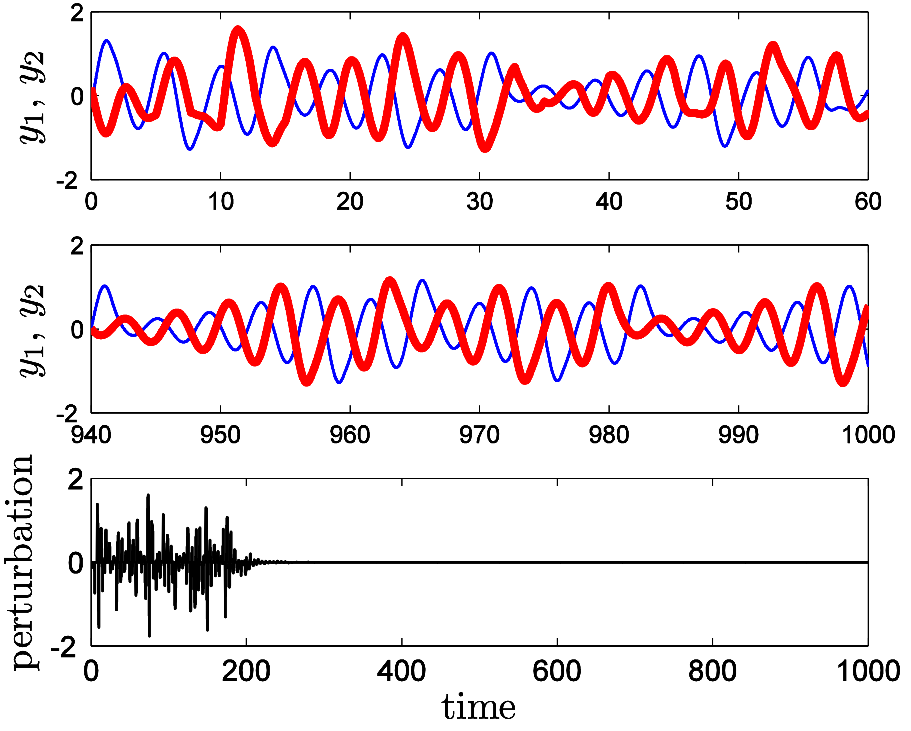


Optimal control strength

$$k_0 = -3.66$$

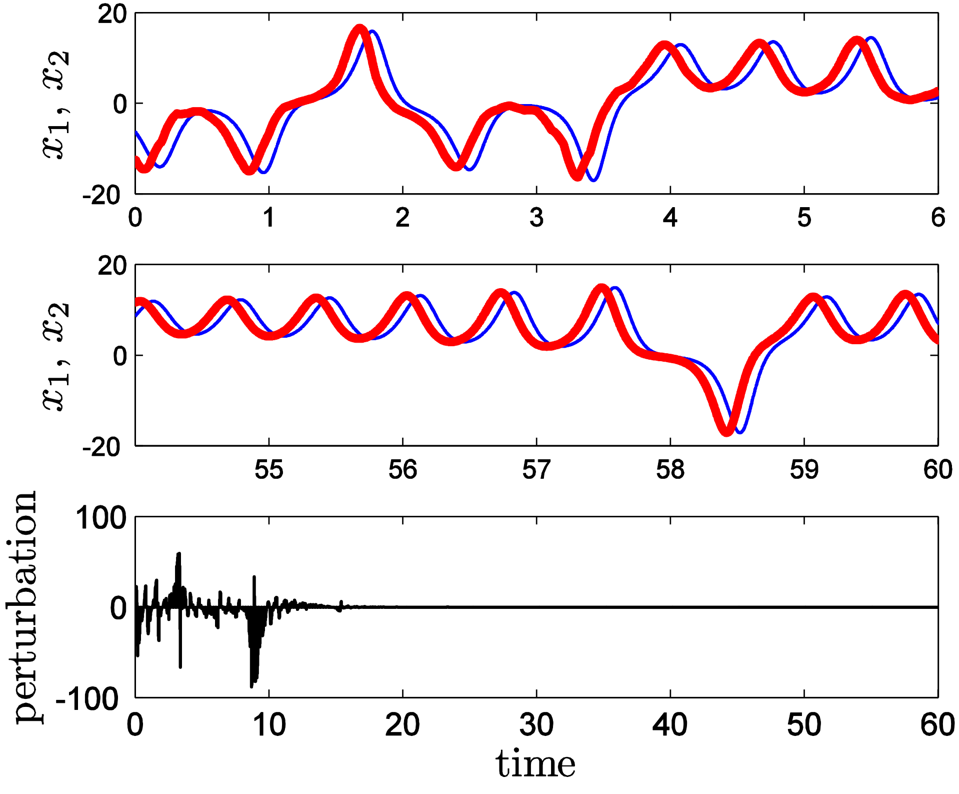
$$\dot{\mathbf{r}}_1(t) = \mathbf{f}\left(\mathbf{r}_1(t)\right) \quad \dot{\mathbf{r}}_2(t) = \mathbf{f}\left(\mathbf{r}_2(t)\right) + u(t)\mathbf{I}$$

The Chua systems



$$\tau = 2.5,\ \theta = 0.2 \qquad k_0 = -3.66$$

The Lorenz systems



$\tau = 0.1,\ \theta = 0.01$

$k_0 = 233$

Conclusions

- The phenomenon of anticipating synchronization under an act-and-wait control has been analyzed for unidirectionally coupled chaotic systems.

- The proposed algorithm works well for chaotic systems and provides a long-term anticipation even when a single input of a drive and a single output for a response system are used.

Thank you

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