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The anticipating synchronization of chaotic systems based on an act-and-wait concept

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Outline

- Context
- Two coupling schemes
- Results
- Conclusions

Anticipating chaotic synchronization

Prediction of chaotic dynamics – Voss (2000)

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \quad \text{a drive system without delay}$$

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{K}[\mathbf{r}_2(t - \tau) - \mathbf{r}_1(t)]$$

$$\mathbf{r}_1 = (x_1, y_1, z_1), \mathbf{r}_2 = (x_2, y_2, z_2)$$

\mathbf{K} – constant coupling matrix

τ

time-delay = anticipation time

Stability

Anticipating manifold

$$\mathbf{r}_2(t) = \mathbf{r}_1(t + \tau)$$

is stable only for some definite values of a time-delay and a coupling matrix

 τ K

Problem

Different coupling schemes were proposed to increase a prediction time

An investigation of the **anticipating manifold stability** encounters **difficulties**

Time-delay terms



The anticipating manifold is defined by the infinite number of Lyapunov exponents

Aim

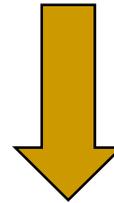
To simplify
an investigation of the stability
of the anticipating manifold for
chaotic systems

Solution

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)),$$

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{K}[\mathbf{r}_2(t - \tau) - \mathbf{r}_1(t) \dots]$$

Act-and-wait control



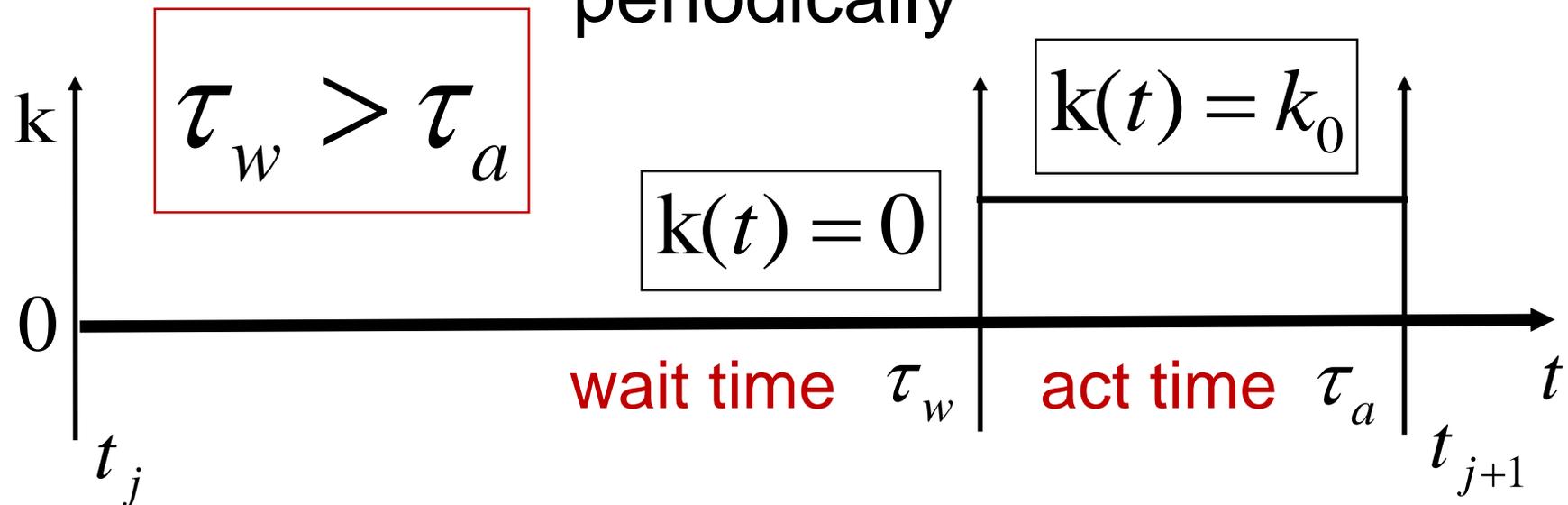
$$k(t)[x_2(t - \tau) - x_1(t) \dots]$$

The anticipating manifold will be defined by the finite number of Lyapunov exponents!

Act-and-wait control concept

$$\dot{\mathbf{r}}(t) = \mathbf{f}(\mathbf{r}(t)) - k(t)\mathbf{F}(t - \tau..)$$

Feedback loop is switched **on** and **off** periodically



$$T = t_{j+1} - t_j = \tau_w + \tau_a$$

Two coupling schemes

Coupling strength: $k(t) = 0,$ wait time

k_0 act time

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)),$$

$$\dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - k(t) \mathbf{F} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

1. Spiral approximate coupling

$$\mathbf{F}^s$$

2. Parabolic approximate coupling

$$\mathbf{F}^p$$

Spiral approximate coupling

$$\tau_w = \tau + \theta, \quad k(t) = 0 \quad T = t_{j+1} - t_j = 2\tau$$

$$\tau_a = \tau - \theta, \quad k(t) = k_0 \quad \theta < \tau$$

$$F^s = x_2(t - \tau) - x_1(t) + a_1^s [x_2(t - \tau - \theta) - x_1(t - \theta)]$$

A weight coefficient a_1^s ?

for the **stable anticipating manifold**

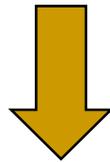
$$\mathbf{r}_2(t) = \mathbf{r}_1(t + \tau)$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \quad \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t) F^s(t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad 10$$

Monodromy matrix

Variation equations for a deviation

$$\delta \mathbf{r}(t) = \mathbf{r}_2(t) - \mathbf{r}_1(t + \tau)$$



$$\delta \ddot{\mathbf{r}}_{j+1} = \mathbf{M} \delta \dot{\mathbf{r}}_j$$

Spiral model

The Rössler system

$$f(r) = [-y - z, x + ay, b + z(x - c)], r = (x, y, z)$$

$$a = 0.15, b = 0.2, c = 10$$

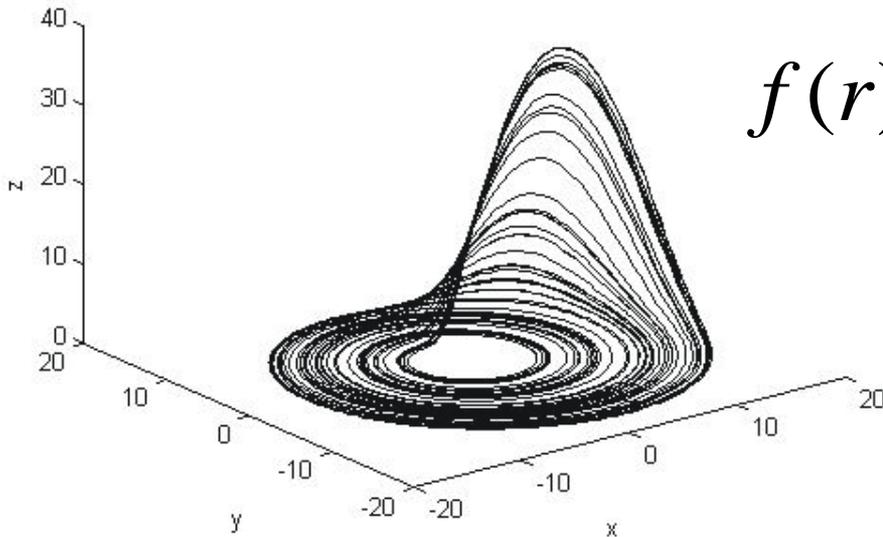
$$\downarrow z = 0$$

Spiral

$$f(r) = [-y, x + ay], r = (x, y)$$

$$\gamma = \frac{a}{2} = 0.075,$$

$$\omega = \sqrt{1 - \gamma^2} = 0.997$$



Spiral approximate coupling

$$x^s(t) = x_0 + Ae^{\gamma t} \cos(\omega t + \varphi_0)$$

$$x(t) = b_0 + b_1 x(t - \tau) + b_2 x(t - \tau - \theta)$$

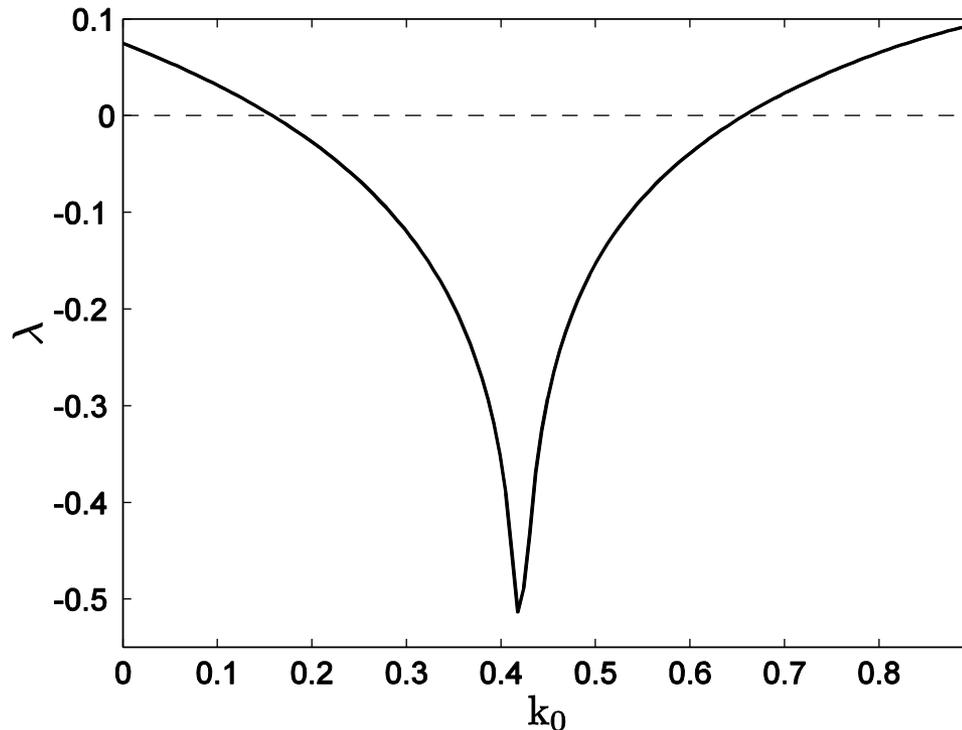
$$F^s = x_2(t - \tau) - x_1(t) + a_1^s [x_2(t - \tau - \theta) - x_1(t - \theta)]$$

$$a_1^s = \frac{e^{\gamma \tau_w} \sin(\omega \tau)}{e^{\gamma \tau} \sin(\omega \tau_w)} \quad \tau_w = \tau + \theta$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t) F^s(t)$$

Two spirals

Floquet exponent



$$\tau = 3, \theta = 0.1$$

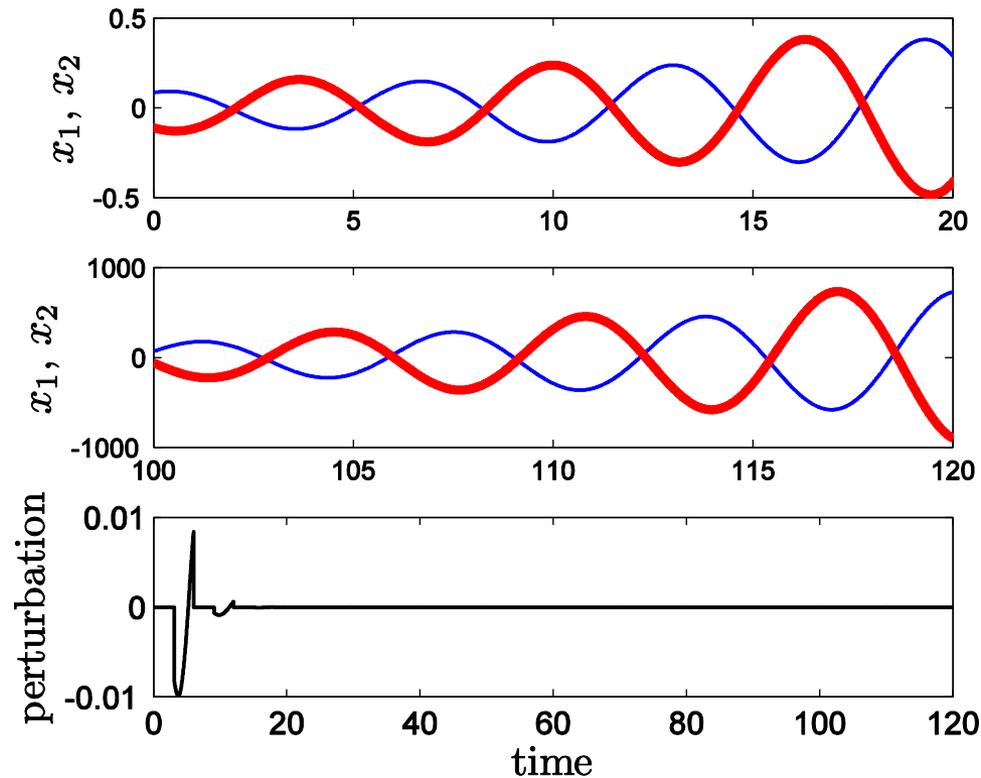
$$k_0 = 0.42$$

Minimum - **optimal control strength**

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - k(t)F^s(t)$$

Two spirals: anticipation

$$\tau = 3, \theta = 0.1 \quad k_0 = 0.42$$



$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t) F^s(t)$$

Parabolic approximate coupling

$$\tau_w = \tau + \theta, \quad k(t) = 0$$

$$\tau_a = \tau - \theta, \quad k(t) = k_0$$

$$T = t_{j+1} - t_j = 2\tau$$

$$F^p = x_2(t - \tau) - x_1(t) + a_1^p [x_2(t - \tau - \theta) - x_1(t - \theta)] \\ + a_2^p [x_2(t - \tau - \theta/2) - x_1(t - \theta/2)]$$

Weight coefficients a_1^p ? a_2^p ?
for the stable anticipating manifold $\mathbf{r}_2(t) = \mathbf{r}_1(t + \tau)$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \quad \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t) F^p(t) \quad 16$$

Weight coefficients

$$x^p(t) = A + Bt + Ct^2$$

$$x(t) = b_0 + b_1x(t - \tau) + b_2x(t - \tau - \theta) + b_3x(t - \tau - \frac{\theta}{2})$$

$$a_1^p = \frac{\tau(2\tau + \theta)}{3\tau\theta + 2\tau^2 + \theta^2},$$

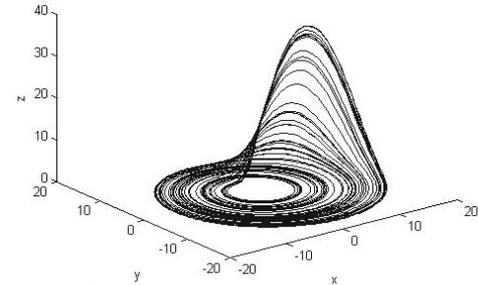
$$a_2^p = -\frac{4\tau\tau_w}{3\tau\theta + 2\tau^2 + \theta^2}$$

$$F^p = x_2(t - \tau) - x_1(t) + a_1^p [x_2(t - \tau - \theta) - x_1(t - \theta)] \\ + a_2^p [x_2(t - \tau - \theta/2) - x_1(t - \theta/2)]$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t)F^p(t)$$

The Rössler systems

Spiral approximate coupling



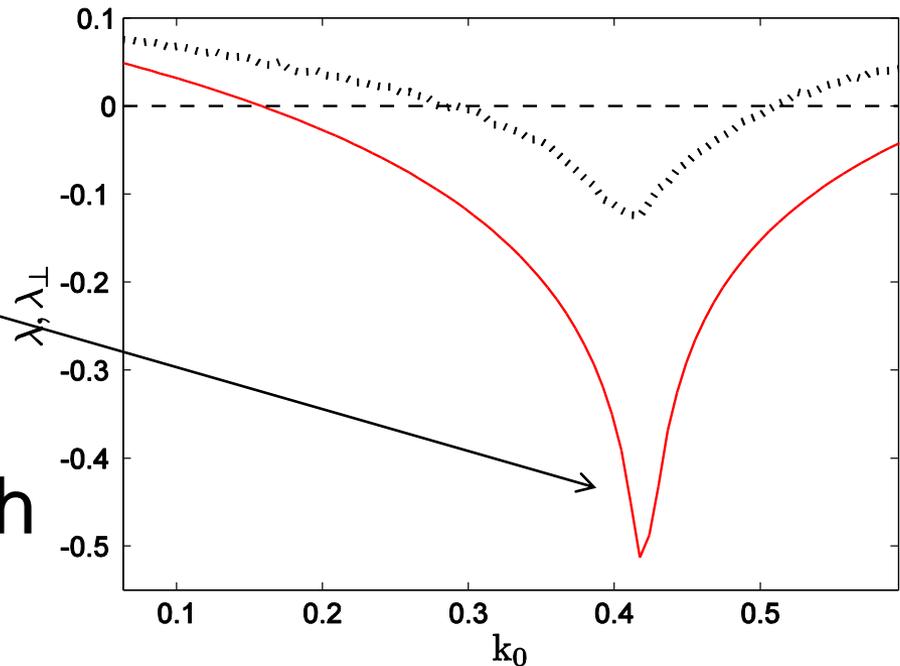
$$F^s = x_2(t-\tau) - x_1(t) + a_1^s [x_2(t-\tau-\theta) - x_1(t-\theta)]$$

$$\tau = 3, \theta = 0.1$$

Spiral systems

Minimum –
optimal control strength

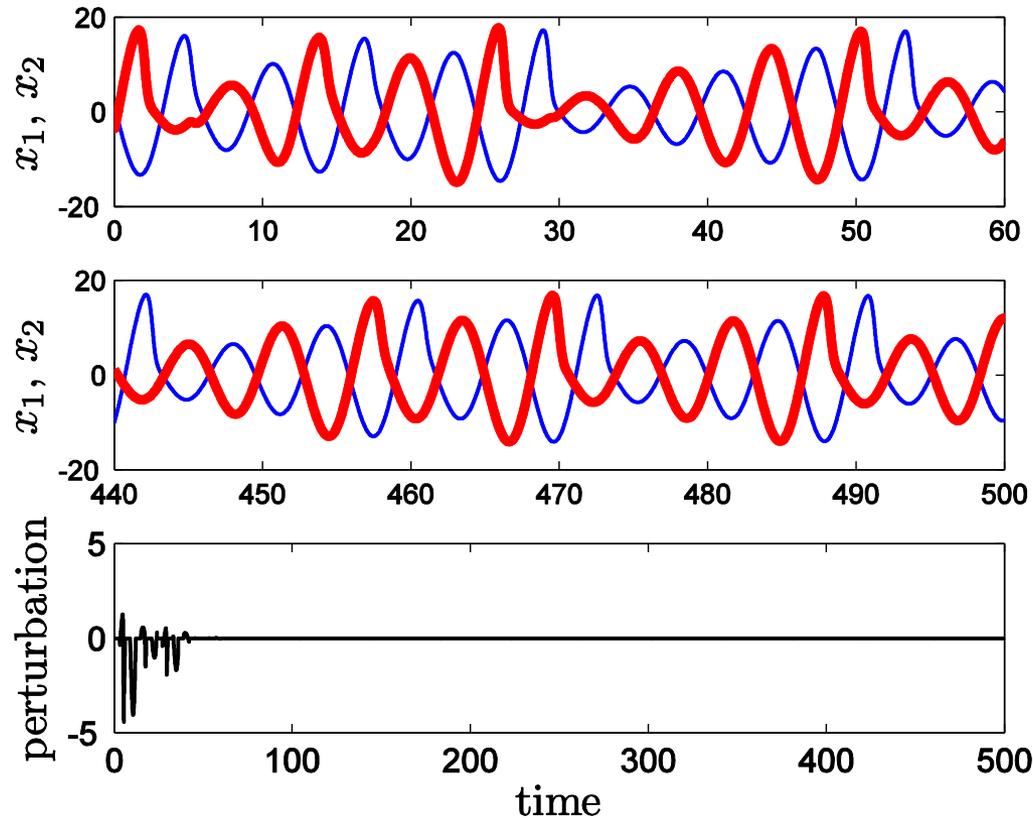
$$k_0 = 0.42$$



$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - k(t)F^s(t)$$

The Rössler systems

Spiral approximate coupling



$$\tau = 3, \theta = 0.1$$

$$k_0 = 0.42$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t)F^s(t),$$

The Rössler systems

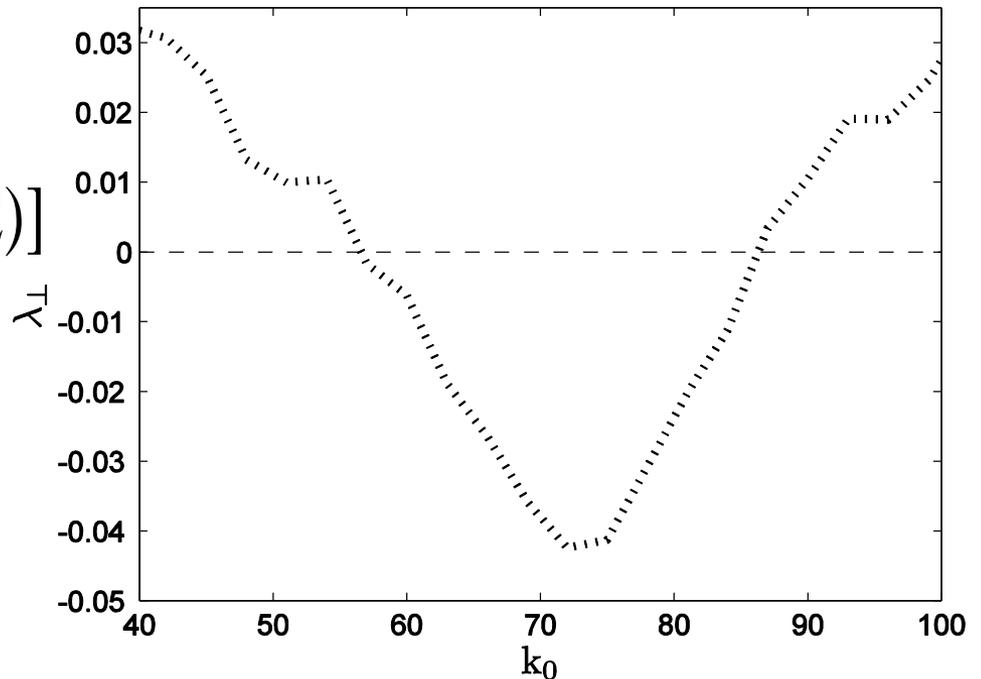
Parabolic approximate coupling

$$F^P = x_2(t - \tau) - x_1(t)$$

$$+ a_1^P [x_2(t - \tau - \theta) - x_1(t - \theta)]$$

$$+ a_2^P [x_2(t - \tau - \theta/2) - x_1(t - \theta/2)]$$

$$\tau = 2.3, \theta = 0.2$$

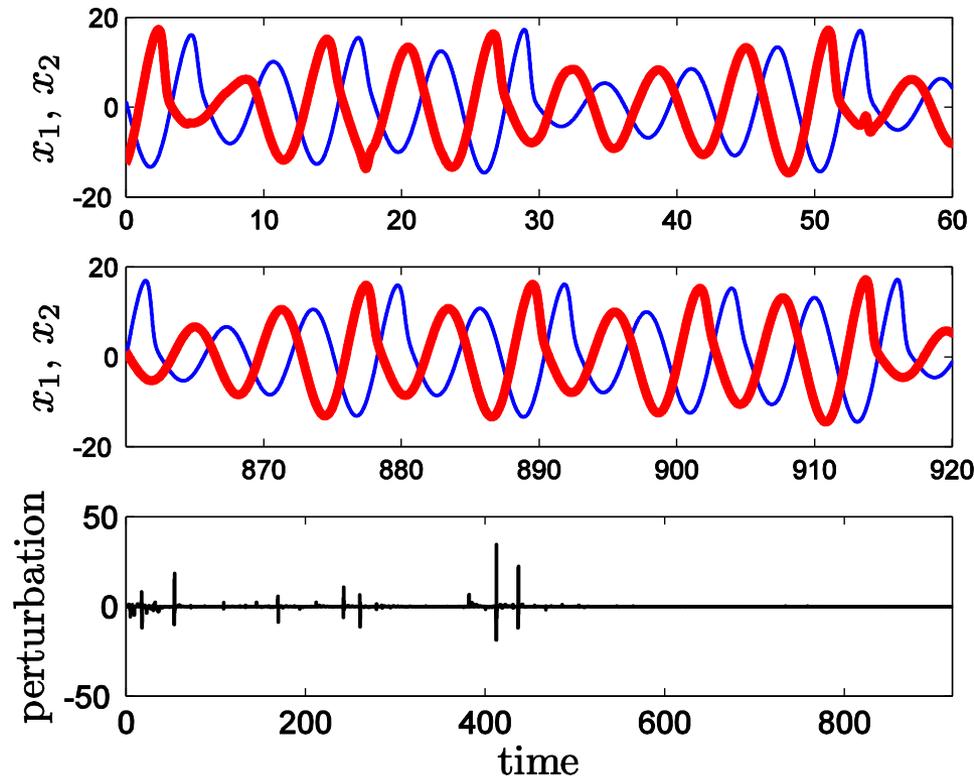


Optimal control strength $k_0 = 72$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t) F^P(t) \quad 20$$

The Rössler systems

Parabolic approximate coupling



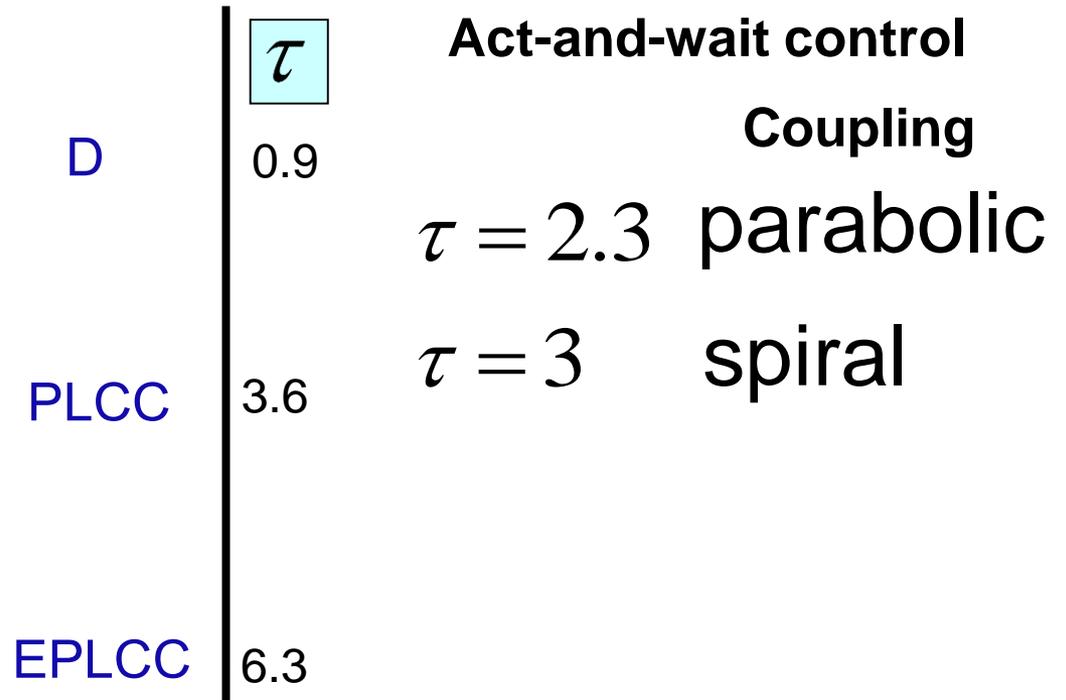
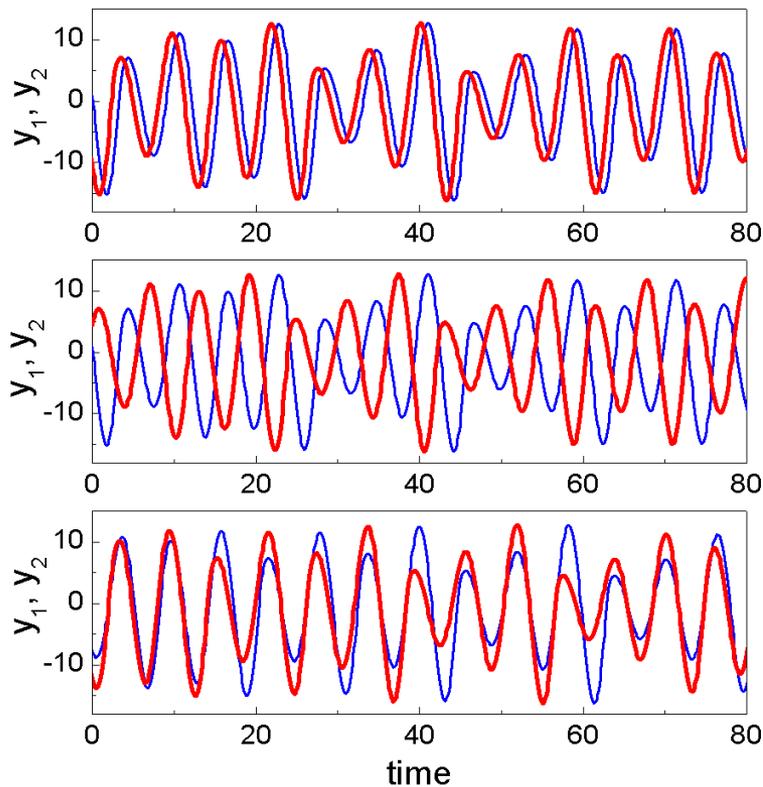
$$k_0 = 72$$

$$\tau = 2.3, \theta = 0.2$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t) F^P(t)_{21}$$

The Rössler systems

Anticipating time



The Chua systems

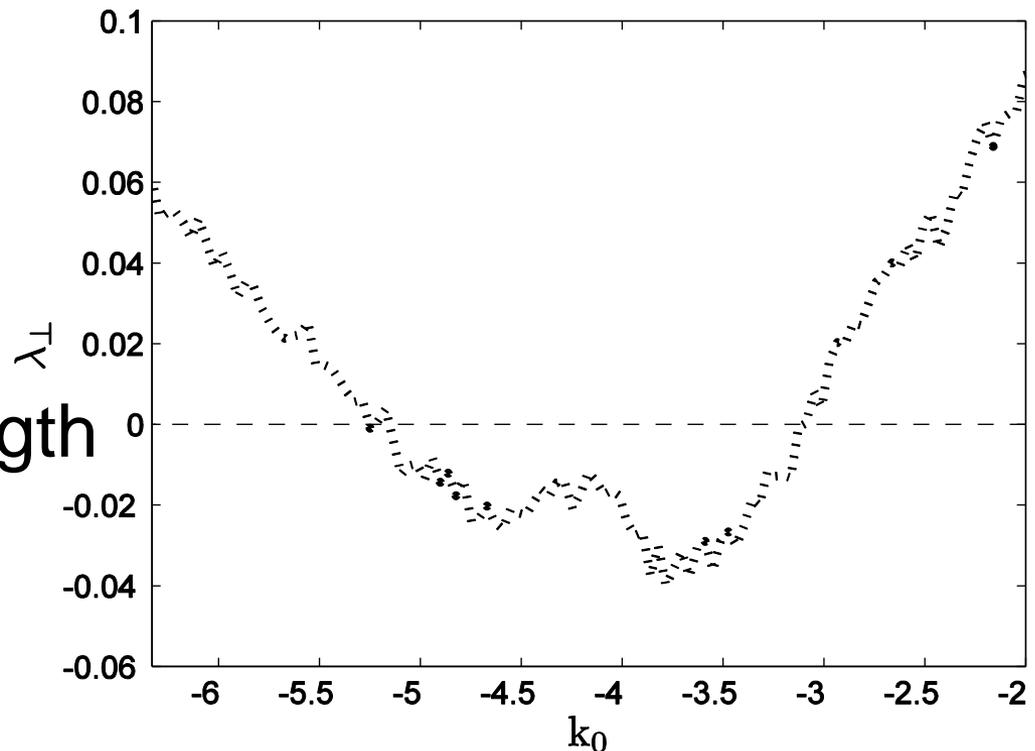
Spiral approximate coupling

$$F^s = x_2(t-\tau) - x_1(t) + a_1^s [x_2(t-\tau-\theta) - x_1(t-\theta)]$$

$$\tau = 2.5, \theta = 0.2$$

Optimal control strength

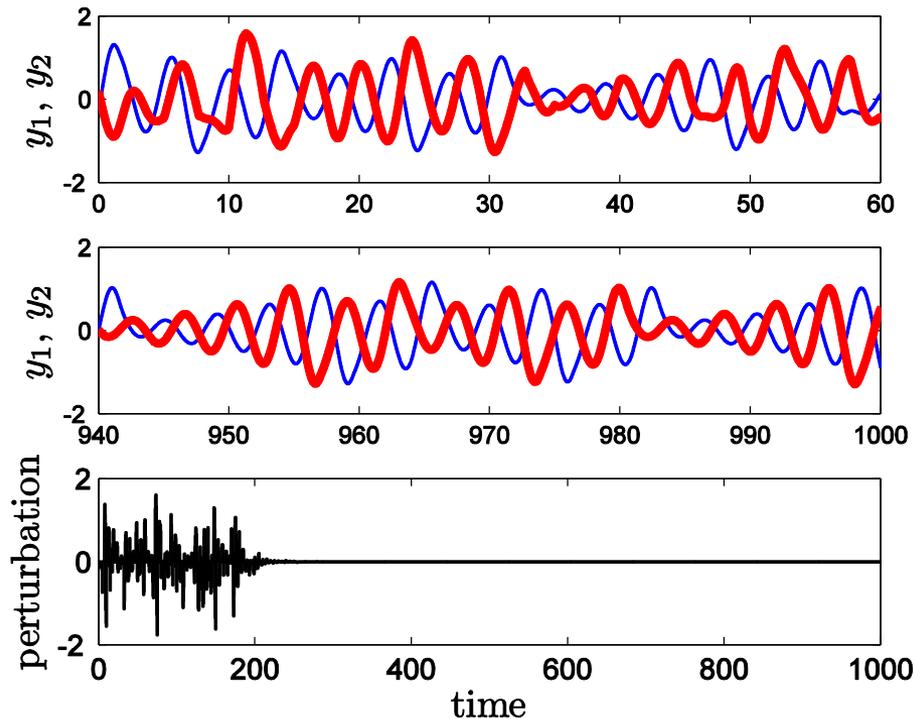
$$k_0 = -3.66$$



$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t)F^s(t)$$

The Chua systems

Spiral approximate coupling

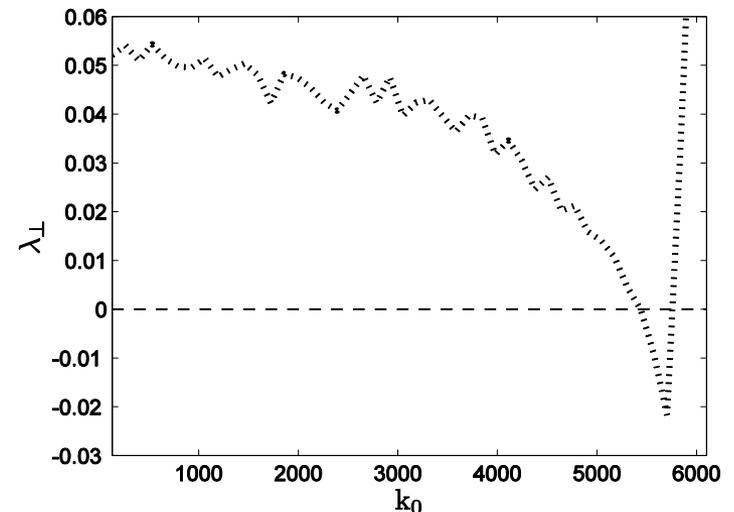
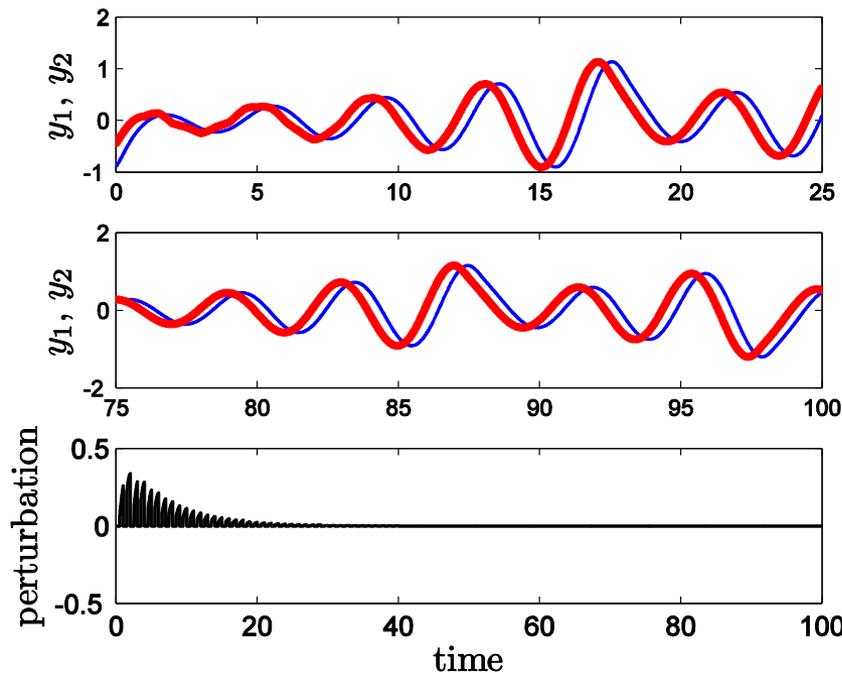


$$\tau = 2.5, \theta = 0.2 \quad k_0 = -3.66$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t)F^s(t)$$

The Chua systems

Parabolic approximate coupling



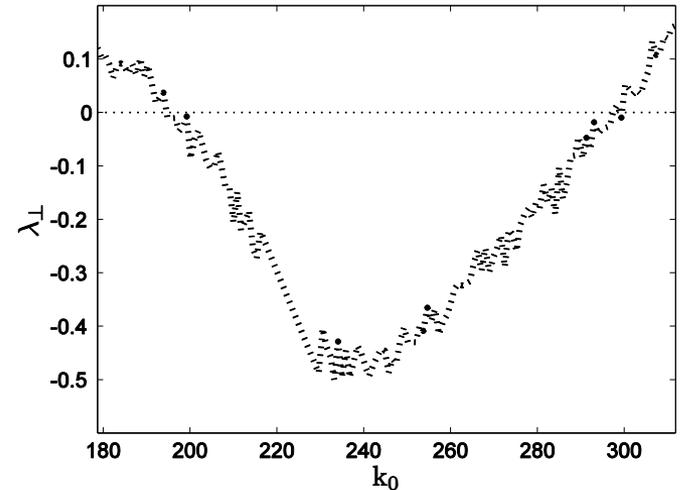
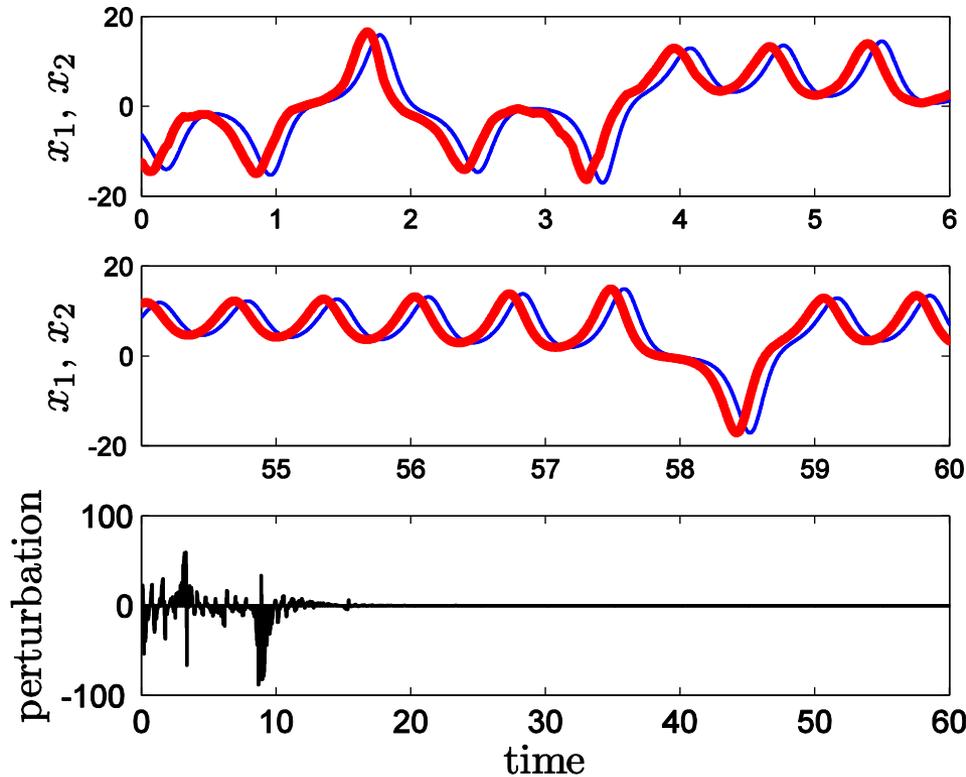
$$\tau = 0.5, \theta = 0.02$$

$$k_0 = 5701$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - k(t) F^P(t)$$

The Lorenz systems

Spiral approximate coupling



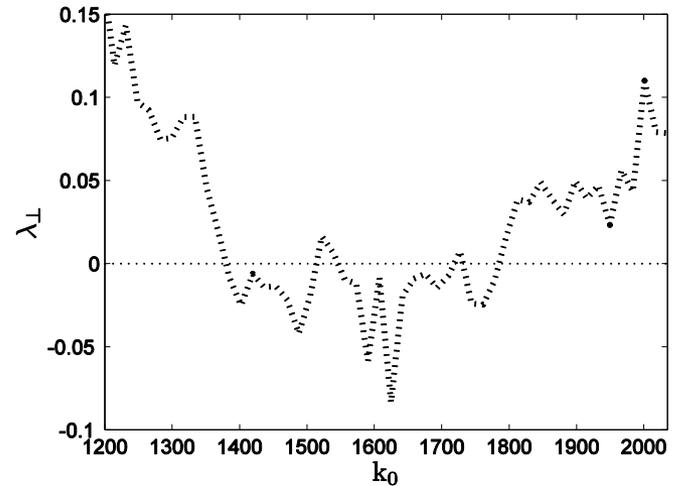
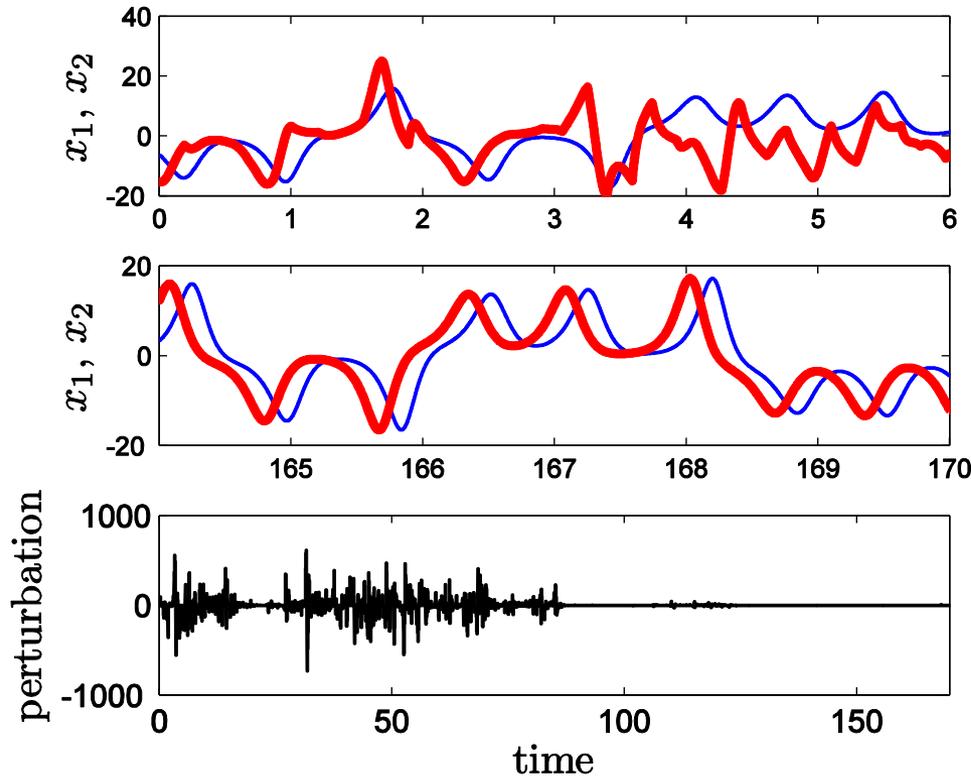
$$k_0 = 233$$

$$\tau = 0.1, \theta = 0.01$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - k(t)F^s(t)$$

The Lorenz systems

Parabolic approximate coupling



$$k_0 = 1624$$

$$\tau = 0.17, \theta = 0.02$$

$$\dot{\mathbf{r}}_1(t) = \mathbf{f}(\mathbf{r}_1(t)), \dot{\mathbf{r}}_2(t) = \mathbf{f}(\mathbf{r}_2(t)) - \mathbf{k}(t) F^P(\mathbf{r}_2(t))$$

Conclusions

- The phenomenon of anticipating synchronization under an act-and-wait control has been analyzed analytically as well numerically in unidirectionally coupled chaotic systems.
- Two coupling schemes with additional time-delays were proposed.
- The proposed algorithm works well for chaotic systems and provides a long-term anticipation even when a single input of a drive and a single output for a response system are used.

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