Controlling synchrony in oscillatory networks via act-and-wait algorithm

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Introduction (I)

- Synchronization of individual neurons is believed to play the crucial role in the emergence of pathological rhythmic brain activity in Parkinson's disease, essential tremor, and epilepsies

- The development of techniques for suppression of the undesired neural synchrony constitutes an important clinical problem

- Electrical deep brain stimulators (DBS) have been developed and implanted in patients to discharge an electrical high-frequency (>100 Hz) signal into the brain tissue and restore the normal activity

- Nevertheless, the mechanism of action of DBS is still unclear. Recently, we have proposed an exploration of the therapeutic effect gained by the high-frequency stimulation in terms of a stabilization of the neuron's resting state. This effect is similar to the stabilization of the upside-down position of a rigid pendulum by vibrating its pivot up and down at a suitably high frequency (Kapitza's pendulum):

K. Pyragas, V. Novičenko and P. Tass, Biol Cybern 107, 669 (2013)

Introduction (II)

- **Shortcoming: Efficiency of the DBS decreases with time due to the adaptation of the brain to stimulation.**

Hence, there is a significant clinical need for less invasive and more effective stimulation techniques!

- Methods for the control of synchronization in oscillatory networks developed to date can be subdivided into two categories: without feedback and with feedback

- **The methods without feedback** do not require any online measurements, however, they require considerably large perturbations

- **The feedback methods** can stabilize the desired unsynchronized state with minimal stimulation intensities

Introduction (III)

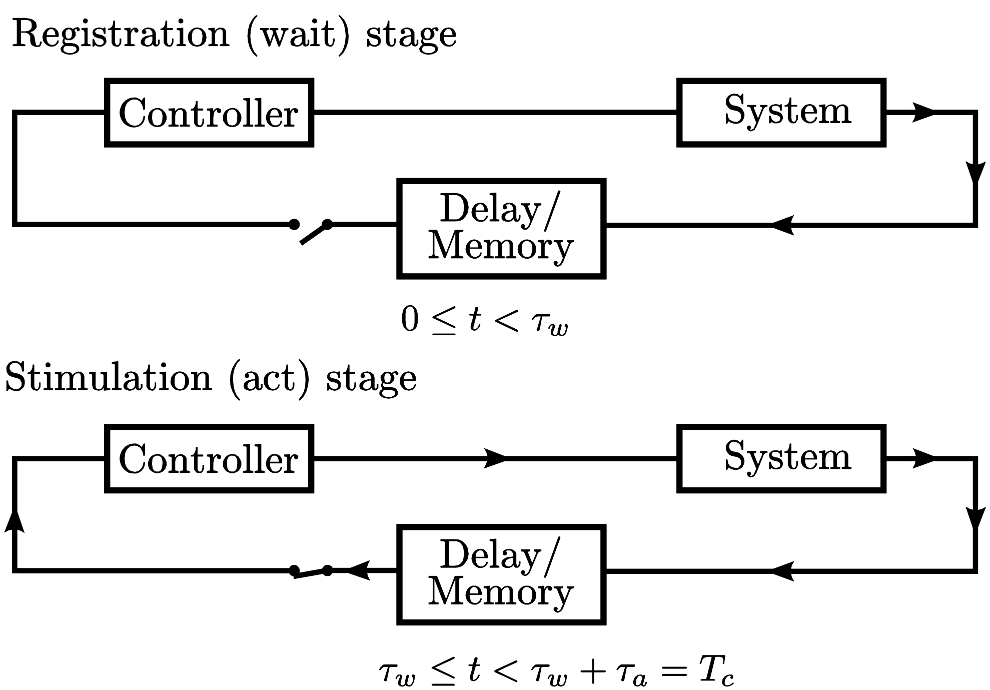
- In DBS the feedback methods meet fundamental problems, since it is impossible to register and stimulate the whole network at the same time. The stimulation current exceeds the measured neuronal currents by six orders of magnitude, so the measurements are corrupted by strong artifacts.

- To avoid this problem a feedback algorithm with a spatially separated stimulation and registration setup has been suggested in our previous publication:

K. Pyragas, O. Popovych and P. Tass, EPL, 80, 40002 (2007)

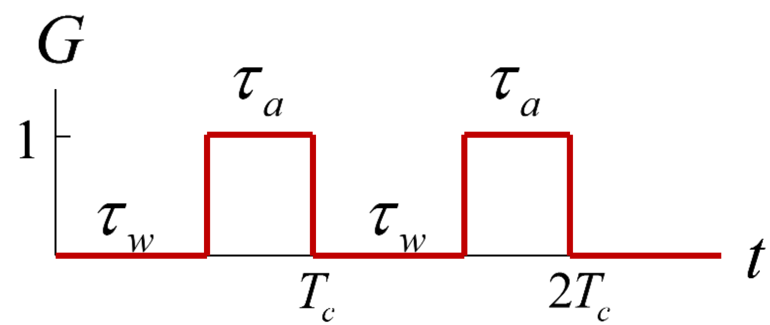
Here we propose an alternative, more efficient way to avoid the above problem by separating the stimulation and registration processes in time rather than in space.

Act-and-wait control algorithm



$$\dot{\vec{x}} = \vec{f}\left(\vec{x}\right) - PG\left(t\right)\vec{x}\left(t-\tau_a\right)$$

$$G\left(t\right) = G\left(t+T_c\right)$$



For $\tau_a \le \tau_w$ the time-delay problem can be treated as the finite dimensional, i.e., the stability of the controlled state is characterized by a finite number of Floquet exponents

T. Insperger, IEEE Transactions on Control Systems Technology 14, 974 (2006).

Globally coupled Landau-Stuart oscillators (II)

Coupling and control via both variables

$$\dot{z}_j = \left(i\omega_j + 1 - \left|z_j\right|^2\right) z_j + KZ - PG(t)Z(t-\tau_a), \qquad j = 1,\dots,N$$

Uncoupled Landau-St oscillators

Global coupling

Act-and wait control

$$Z = \frac{1}{N}\sum_{k=1}^{N} z_k$$

Reduction to the Kuramoto-type model:

$$z_j = \rho_j e^{i\theta_j}$$

$$\left(\rho_j \approx 1\right)$$

$$\dot{\theta}_j = \omega_j + \left[Kr - PG(t)r_\tau\right]e^{-i\theta_j} / 2i + c.c.$$

$$r = \frac{1}{N}\sum_{k=1}^{N} e^{i\theta_k}$$ – **order parameter**

$$r_\tau \equiv r(t-\tau)$$

$$\tau_a = \tau_w = \tau$$

Globally coupled Landau-Stuart oscillators (III)

Thermodinamic limit $N \to \infty$:

$$\frac{\partial}{\partial t} f(\omega,\theta,t) = -\frac{\partial}{\partial \theta}\left[f(\omega,\theta,t) v(\omega,\theta,t) \right]$$ **– continuity equation**

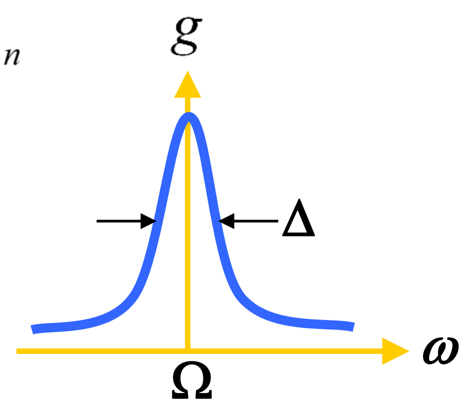
$$v(\omega,\theta,t) = \omega + \left[Kr - PG(t) r_\tau \right] e^{-i\theta} / 2i + c.c.$$ **– velocity**

$$r = \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} d\omega e^{i\theta} f(\omega,\theta,t)$$ **– order parameter**

Analytical treatment using Ott-Antonsen (2008) anzatz:

$$f = \frac{g(\omega)}{2\pi}\left(1 + \left[\sum_{n=1}^{\infty} f_n(\omega,t) + c.c.\right]\right), \quad f_n(\omega,t) = \left[\alpha(\omega,t)\right]^n$$

$$g(\omega) = (\Delta/\pi)\left[(\omega-\Omega)^2 + \Delta^2\right]^{-1}$$ **- Lorentz distribution**

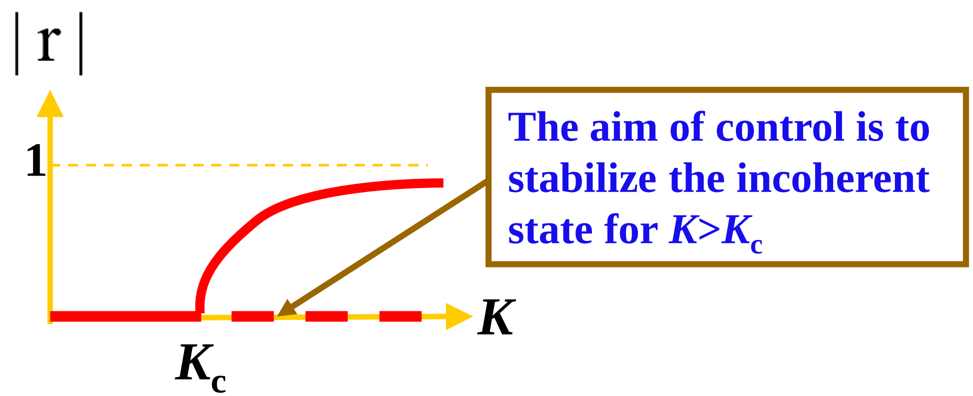


Globally coupled Landau-Stuart oscillators (IV)

Closed equation for the order parameter:

$$\dot{r} = \left[i\Omega - \Delta + \frac{K}{2}\left(1 - |r|^2\right)\right] r + \frac{G(t)}{2}\left(P^* r^2 r_\tau^* - \mathrm{P} r_\tau\right)$$

Without control (*P*=0) we have the classical Kuramoto model:



$$K_c = 2\Delta$$ **– synchronization threshold**

Globally coupled Landau-Stuart oscillators (V)

Linear stability of the incoherent state $r=0$ in the presence of control:

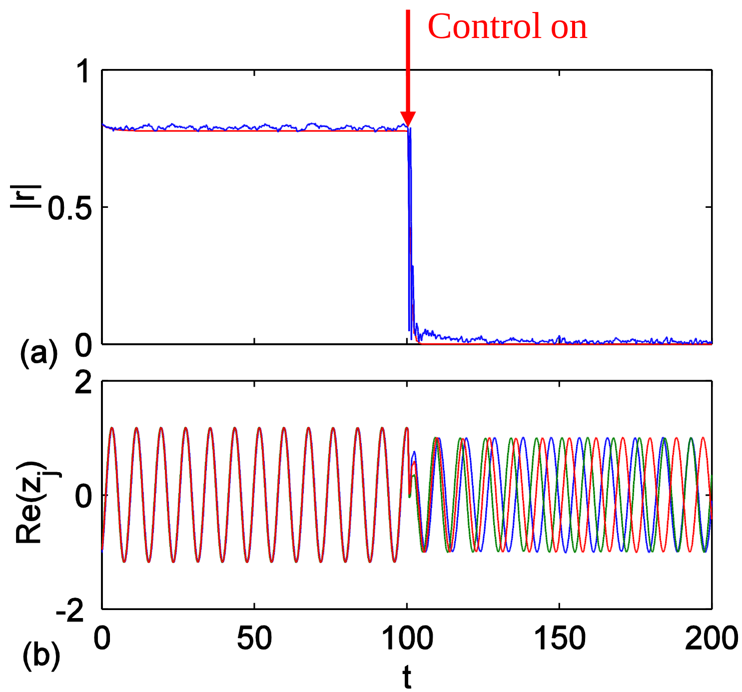
$$\delta\dot{r}(t)=(K/2-\Delta)\delta r(t)-G(t)P\delta r(t-\tau)$$

- This equation contains time-delay term, which is usually associated with an infinite-dimensional phase space and thus with infinite number of eigenvalues.
- However, due to the presence of the periodic act-and-wait function $G(t)$ it can be reduces to a two-dimensional map, whose stability is defined by only two Floquet multiliers $\mu_{1,2}$
- These two multipliers can be easily placed inside the unite circle of the complex plain ($|\mu_{1,2}|<1$) by appropriate choice of the complex coupling strength P:

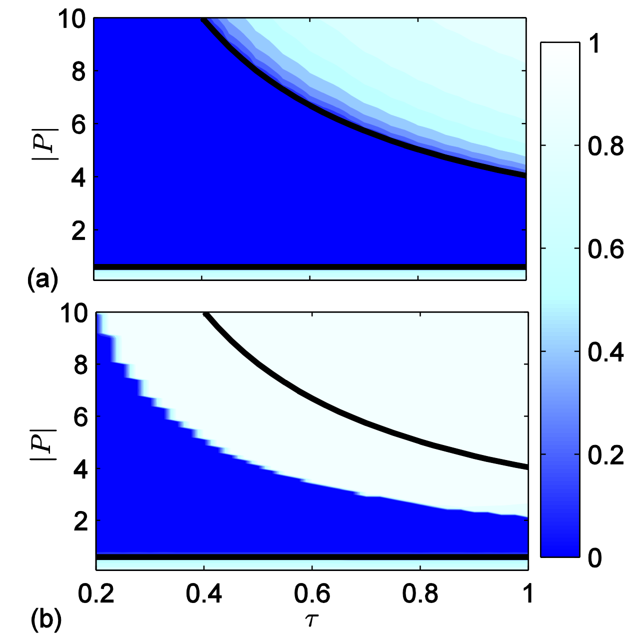
$$\frac{2}{\tau}\left[e^{(K/2-\Delta)\tau}-e^{-(K/2-\Delta)\tau}\right]<|P|<\frac{2}{\tau}\left[e^{(K/2-\Delta)\tau}+e^{-(K/2-\Delta)\tau}\right]$$

Globally coupled Landau-Stuart oscillators (VI)

Numerical demonstrations:



Dynamics of N=1000 coupled Landau-Stuart oscillators



Incoherent state stability domains. The color encodes the value of the order parameter

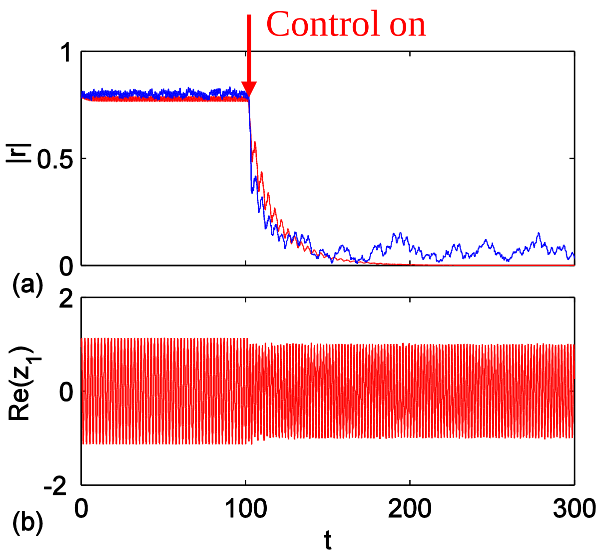
Globally coupled Landau-Stuart oscillators (VII)

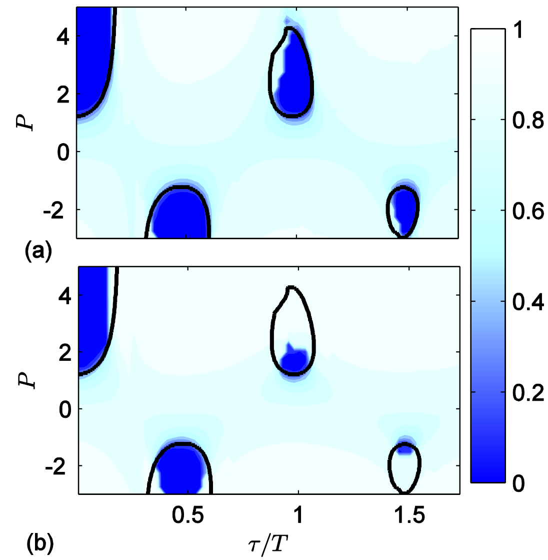
Coupling and control via a single variable

$$\dot{z}_j = \left(i\omega_j + 1 - \left|z_j\right|^2\right) z_j + K \operatorname{Re} Z - PG(t) \operatorname{Re} Z(t-\tau)$$

$$\dot{\theta}_j = \omega_j - \sin(\theta_j) \operatorname{Re}\left[Kr - PG(t) r_\tau\right]$$ ⟸ **Phase dynamics**

$$\dot{r} = (i\Omega - \Delta) r + (1 - r^2)\left[K \operatorname{Re}(r) - G(t) P \operatorname{Re}(r_\tau)\right]/2$$ ⟸ **Order parameter**





$T = 2\pi/\Omega$

Synaptically coupled FitzHug-Nagumo neurons (I)

$$\dot{v}_j = v_j - v_j^3/3 - w_j + I_j - I_{syn} - I_{con}$$
$$\dot{w}_j = \varepsilon\left(v_j + \beta - \gamma w_j\right)$$

$$j = 1, \ldots, N$$

$$I_{syn} = g\left(v_j - v_c\right)\frac{1}{N-1}\sum_{k \neq j}\Theta\left(v_k - v_0\right), \quad \Theta(v) = \frac{1}{1 + e^{-v/v_{th}}}$$

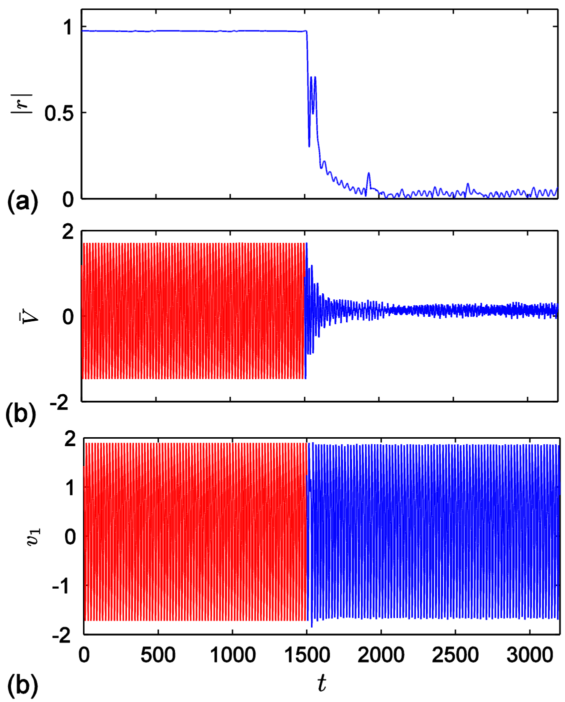
$$I_{con}(t) = G(t)PV(t-\tau), \quad V(t) = \frac{1}{N}\sum_{k=1}^{N} v_k(t)$$

Charge-balanced stimulation:

$$I_{con}(t) = G(t)P\left[V(t-\tau) - V_n\right], \quad V_n = \frac{1}{\tau}\int_{2n\tau}^{(2n+1)\tau} V(t)$$

Synaptically coupled FitzHug-Nagumo neurons (II)

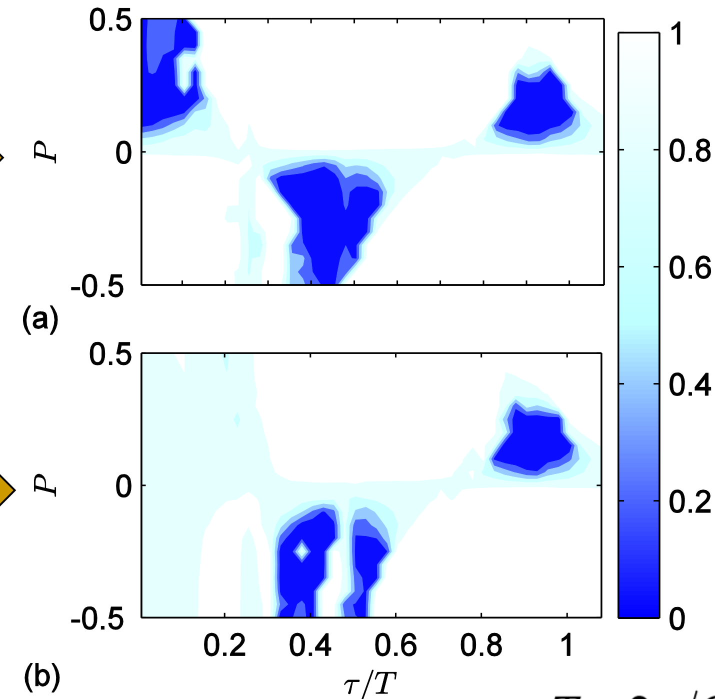
Numerical demonstrations:



Dynamics of N=500 synaptically coupled FitzHug-Nagumo neurons

Withot charge balance

Charge-balanced



$$T = 2\pi/\Omega$$

Incoherent state stability domains

Synaptically coupled Hodgkin-Huxley neurons under charge-balanced act-and-wait control (I)

$$C\dot{v}_j = -gN_a m_j^3 h_j \left(v_j - v_{Na}\right) - g_K n_j^4 \left(v_j - v_K\right) - g_L \left(v_j - v_L\right) + I_j + I_{syn} + I_{con}$$

$$\dot{m}_j = \alpha_m\left(v_j\right)\left(1 - m_j\right) - \beta_m\left(v_j\right) m_j$$

$$\dot{h}_j = \alpha_h\left(v_j\right)\left(1 - h_j\right) - \beta_h\left(v_j\right) h_j$$

$$\dot{n}_j = \alpha_n\left(v_j\right)\left(1 - n_j\right) - \beta_n\left(v_j\right) n_j$$

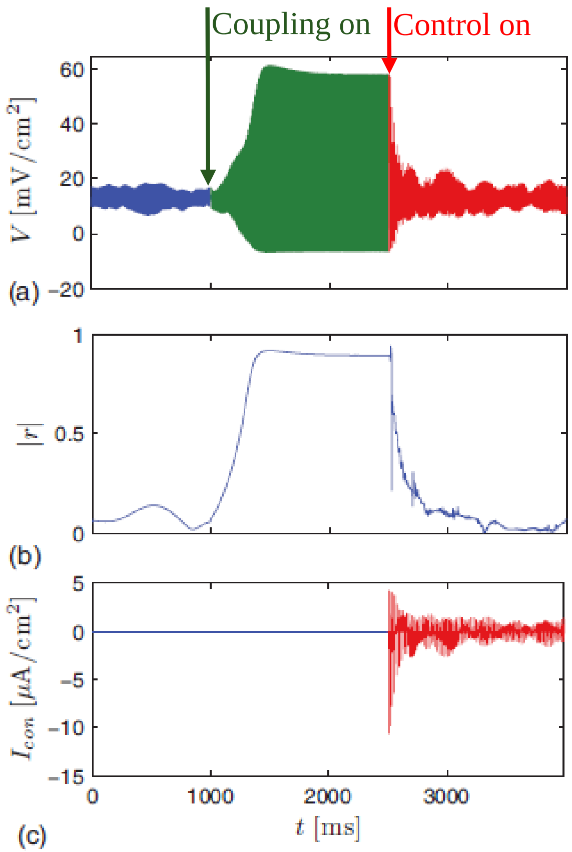
$$j = 1, \ldots, N$$

$$I_{syn} = g\left(v_j - v_c\right)\frac{1}{N-1}\sum_{k \neq j}\Theta\left(v_k - v_0\right), \quad \Theta\left(v\right) = \frac{1}{1 + e^{-v/v_{th}}}$$

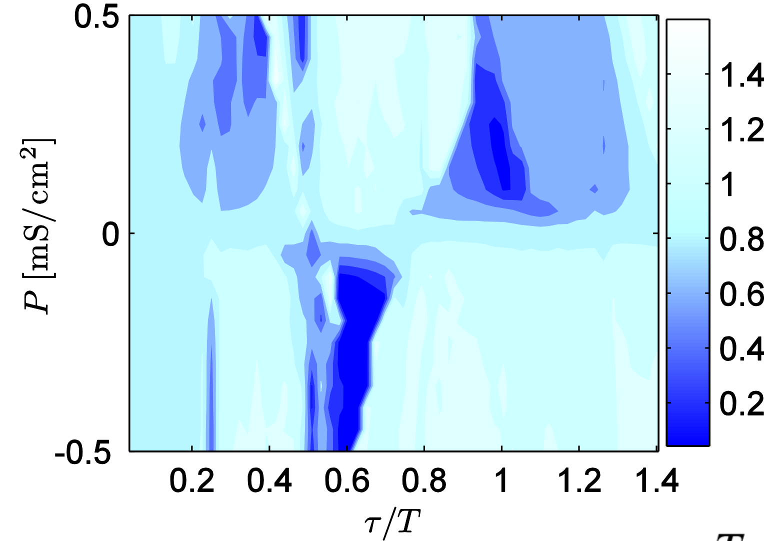
$$I_{con}\left(t\right) = G\left(t\right) P\left[V\left(t - \tau\right) - V_n\right], \quad V_n = \frac{1}{\tau}\int_{2n\tau}^{(2n+1)\tau} V\left(t\right)$$

Synaptically coupled Hodgkin-Huxley neurons under charge-balanced act-and-wait control (II)

Numerical demonstrations:



Dynamics of N=100 synaptically coupled Hodgkin-Huxley neurons



$$T = 2\pi/\Omega$$

Incoherent state stability domains

Conclusions

- The act-and-wait algorithm allows an effective suppression of synchrony in ensembles of globally coupled oscillators in the situation when the simultaneous stimulation and registration of the system is impossible
- The time delayed feedback with the periodically switched on and off control gain may be superior to the time delayed feedback with the constant gain, since the stability of the controlled system is defined by the finite number of the Floquet exponents
- Our algorithm satisfies the charge-balanced requirement, which is clinically mandatory to avoid tissue
- The act-and-wait algorithm is robust; it works for neural networks constructed from different neurons

Details in: I. Ratas and K. Pyragas, Phys. Rev. E, in press (2014)

Acknowledgments

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