Time-Delayed Feedback Control Design Beyond the Odd-Number Limitation

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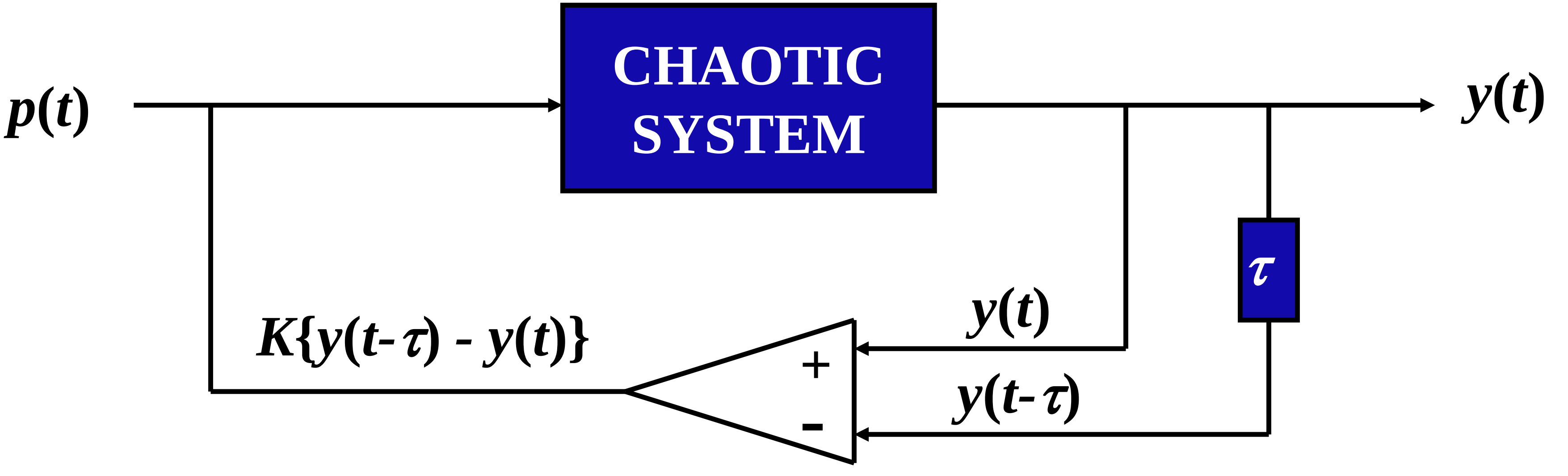
Outline

- Introduction to time-delayed feedback control (TDFC) algorithm
- Odd number limitation (Nakajima's theorem)
- TDFC modifications to bypass the limitation
- Refuting the limitation for autonomous systems (Fiedler et al.)
- Corrected version of the odd number theorem (Hooton, Amann)
- Explicit form for Hooton's and Amann's criterion
- Control matrix design
- Numerical examples (Lorenz and Chua systems)
- Conclusions

Delayed feedback control algorithm

K. Pyragas, Phys. Lett. A 170, 421 (1992)



$$\dot{\vec{x}} = \vec{f}(\vec{x}, p)$$

$$y(t) = g(\vec{x})$$

$$p = p(t) = K[y(t) - y(t - \tau)]$$

τ – period of unstable periodic orbit

Experimental implementations (I)

Electronic chaos oscillators

- Pyragas, Tamaševičius (1993)
- Gauthier et al. (1994)
- Kittel et. al. (1994)
- Celka (1994)
- Socolar et al. (1994)
- Benner et al. (1997)
- Sukow et al. (1997)
- Chang et al. (1998)
- Just et al. (1999, 2000)
- Loewenich et al. (2004)
- Boccaletti et al. (2004)
- Choe et al. (2005)
- Ahlborn, Parlitz (2006)
- Hoehne et al. (2007)
- Tamaševičius et al. (2007)
- Loewenich et al. (2010)

Lasers

- Belawski et al. (1994)
- Erneux et al. (1995)
- Lu, Yu, Harrison (1998)
- Arecchi et al. (2002)
- Boccaletti et al. (2004)
- Bielawski et al. (2005)
- Schikora et al. (2006, 2008)
- Dahms et al. (2010)
- Schicora at al. (2011)

Chemical systems

- Schneider et al. (1993)
- Parmananda et al. (1999)
- Tsui, Jones (2000)
- Kiss et al. (2000)
- Kim et al. (2001)
- Bertram et al. (2003)
- Beta et al. (2003)
- Kiss et al. (2006, 2008)

Experimental implementations (II)

Mechanical pendulums

- **Hikihara, Kawagoshi (1996)**
- **Christini et al. (1997)**
- **Sieber et al. (2008)**

Gas discharge systems

- **Pierre et al. (1996)**
- **Mausbach et al. (1997)**
- **Wei et al. (2004)**

Plasma

- **Gravier et al. (1999)**
- **Fukuyama et al. (2002)**
- **Fukuyama et al. (2006)**

Ferromagnetic resonance

- **Ye et al. (1995)**
- **Benner et al. (1997)**

DC-DC boost converter

- **Natsheh et al. (2009)**

Walking control of robots

- **Sugimoto, Osuka (2002)**

Cardiac systems

- **Hall et al. (1997)**
- **Rappel et al. (1999)**
- **Berger et al. (2007)**

Chaotic Tailor-Couette flow

- **Lüthje et al. (2001)**

Atomic force microscope

- **Yamasue et al. (2009)**

Control of bifurcations

- **Sieber et al. (2008)**
- **Barton and Sieber (2013)**

Odd number limitation (ONL)

Ushio (1996), Nakajima and Ueda (1997,1998), Just at al. (1997)

Nakajima‘s theorem (1997):

The TDFC cannot stabilize any periodic orbit with an odd number of real Floquet multipliers larger than unity

TDFC modifications to bypass the limitation

- **Oscillating delayed feedback**

  Schuster and Stemmler, PRE (1997)

- **Half-period delayed feedback (symmetric orbits)**

  Nakajima and Ueda, PRE (1998)

- **Unstable controller**

  K. Pyragas, PRL (2001)
  K. Pyragas, V. Pyragas, and H. Benner, PRE (2004)
  V. Pyragas and K. Pyragas, PRE (2006)
  A. Tamasevicius, G. Mykolaitis, V. Pyragas, and K. Pyragas, PRE (2007)
  K. Hoehne, H. Shirahama, C. Choe, H. Benner, K. Pyragas, W. Just, PRL (2007)

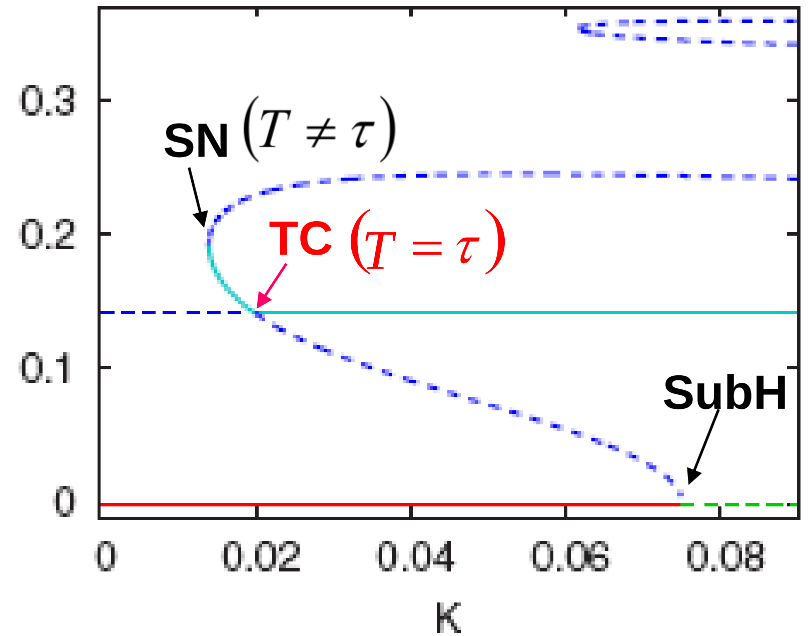
Counterexample refuting the odd number limitation

Fiedler at al., PRL (2007) → **Refuting ONL for autonomous systems**

Counterexample: Normal form of SubH bifurcation under TDFC

$$\dot{z}(t) = \left[(\lambda + i) + (1 + i\gamma)|z(t)|^2\right] z(t) - Ke^{-i\beta}\left[z(t) - z(t-\tau)\right]$$

Complex gain (rotaiting feedback)



- **TDFC induces a pair of orbits (a saddle-node bifurcation) with** $T \neq \tau$

- **The UPO coalesces with iduced orbit at a transcritical bifurcation and exchange the stability**

The main goal of our research

The Fieddler’s et al. idea has been succesfully aplied to:

- **The Lorenz system close to subcritical Hopf bifurcation**
  Postlethwaite and Silber, PRE (2007)
- **Laser experiment close to subcritical Hopf bifurcation**
  Schikora, Wuensche and Henneberger, PRE (2011)
- **Rotating waves near a fold bifurcation**
  Fiedler et al. , PRE (2011)

The main goal of our research is the control matrix design far from bifurcation points of periodic orbits

Corrected version of the odd number theorem

Hooton and Amann, PRL (2012)

$$\dot{\vec{x}}(t) = \vec{f}(\vec{x}(t)) + K[\vec{x}(t-\tau) - \vec{x}(t)]$$

$$\vec{x}(t) = \vec{\xi}(t)$$ ⇐ UPO

An UPO of autonomous sysytem cannot be stabilized by TDFC if:

$$(-1)^m \beta < 0$$

$$\beta = \lim_{\tau \to T} \frac{\tau - T}{\tau - \Theta(K, \tau)}$$

m - the number of real Floquet multipliers larger than 1

T - the period of UPO

τ - slightly mismatched delay time ($\tau \neq T$)

$\Theta(K, \tau)$ - the period of the induced orbit at the mismatched delay

β<0 – necessary condition (for m odd) for the stabilization of UPO

An explicit form for Hooton’s and Amann’s criterion

Phase reduction theory approach

V. Novičenko and K. Pyragas, Physica D (2012); PRE (2012)

$$\Theta(K,\tau)=T-(\tau-T)[\alpha(K)-1]+O\left((\tau-T)^2\right)$$

c.f. W. Just et al. PRL (1998) for the case of scalar input

$$\vec{z}(t)=\alpha(K)\vec{\rho}(t)$$

PRC of controlled system

PRC of uncontrolled system

$$\beta=\alpha^{-1}(K)$$

The profile of the PRC is independent of K

Hooton’s and Amann’s necessary condition $\beta<0$:

$$\beta=\alpha^{-1}=1+\sum_{ij}^{n}K_{ij}C_{ij}<0$$

$$C_{ij}=\int_0^T\rho_i\dot{\xi}_j dt$$

$$\dot{\vec{\rho}}^T(t)=-\vec{\rho}^T(t)A_0(t)$$

$$\vec{\rho}^T(t)\vec{\xi}(t)=1$$

Control matrix design

$K = \kappa B$

B has at least one element equael to 1 or -1 and other elements are in the interval [-1 1].

Hooton's and Amann's necessary condition β<0:

$$\kappa > \kappa^* = -\left(\sum_{ij}^{n} B_{ij} C_{ij}\right)^{-1}$$

Other conditions we derive using the relationship between FEs of the TDFC (λ) and PFC (Λ) systems [Pyragas, PRE (2002)]:

TDFC: $\dot{\vec{x}}(t) = \vec{f}(\vec{x}(t)) + \kappa B[\vec{x}(t-\tau) - \vec{x}(t)]$

PFC: $\dot{\vec{x}}(t) = \vec{f}(\vec{x}(t)) + gB[\vec{\xi}(t) - \vec{x}(t)]$

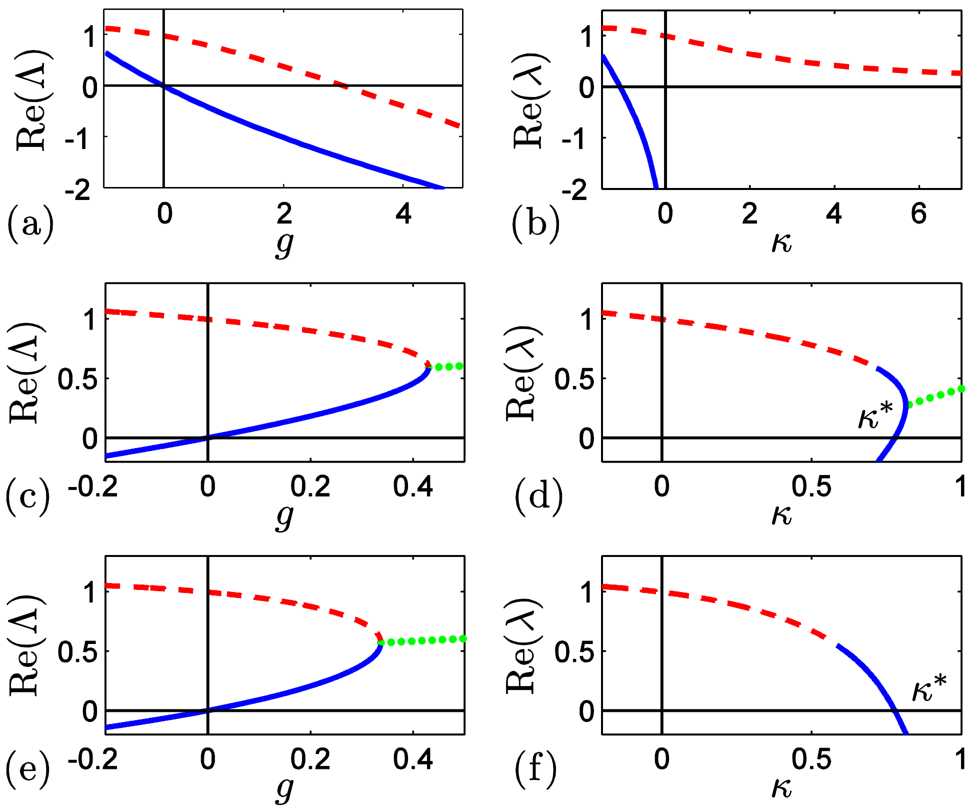
$$\lambda = \Lambda(g)$$

$$\kappa = g[1 - \exp(-\Lambda(g)T)]^{-1}$$

Example: Lorenz system

$$\vec{f}(\vec{x}) = [10(x_2 - x_1),\ x_1(28 - x_3) - x_2,\ x_1x_2 - 8/3x_3]^T$$

PFC: **TDFC:**



$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0.3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Lambda(g)T = ag + bg^2 + O(g^3)$$

$$\kappa^* = a^{-1}$$

$$a = -\sum_{ij}^{n} B_{ij} C_{ij}$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0.5 \\ 0 & 0 & 0 \end{pmatrix}$$

Potentially successful !!!

Additional condition: $\left.\frac{d\lambda}{d\kappa}\right|_{\kappa=\kappa^*} < 0$

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$$\left.\frac{d\lambda}{d\kappa}\right|_{\kappa=\kappa^*} < 0$$

$$\lambda = \Lambda(g)$$
$$\kappa = g\left[1-\exp(-\Lambda(g)T)\right]^{-1}$$

$$\Lambda(g)T = ag + bg^2 + O(g^3)$$

$$1 - 2b/a^2 < 0$$

$$b = \sum_{ijkl}^{n} B_{ij} B_{kl} D_{ijkl}$$

$$D_{ijkl} = D_{klij}$$

The coefficients D_{ijkl} can be estimated from the dependence $\Lambda(g)$ of the trivial FE for small g.

Summary of the algorithm

i. Choose the structure of the matrix *B* with only several nonzero elements in such a way as to make possible the coalescence of the positive and trivial Floquet branches of the PFC system

ii. For the given structure of the matrix *B*, estimate the relevant coefficients C_{ij} and D_{ijkl}

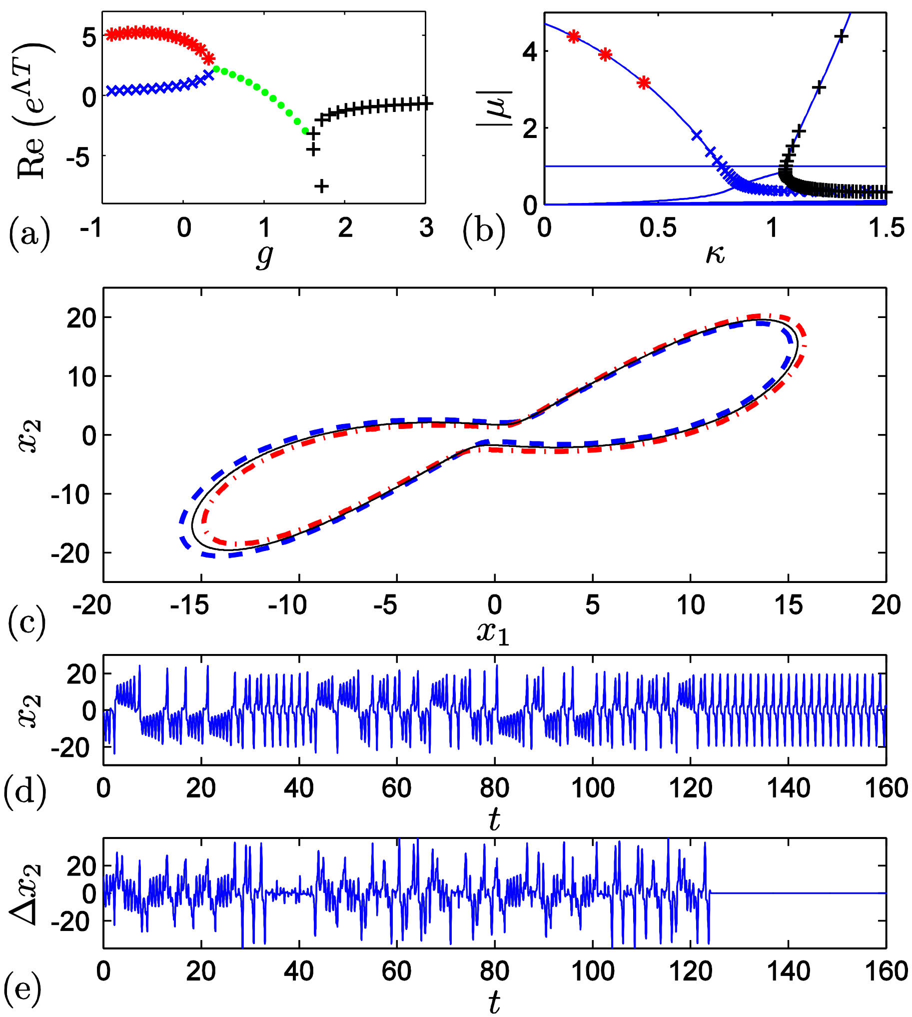
iii. Choose the values of nonzero elements of the matrix B such as to satisfy condition $1-2b/a^2<0$

iv. Compute the threshold κ^* and satisfy condition $\kappa > \kappa^*$

The algorithm considers only most important (real-valued) branches of the FMs and its final outcome has to be verified by more detailed analysis of the stability of the TDFC system

Application to the Lorenz system

PFC: **TDFC:**



$$\dot{\vec{x}}(t) = \vec{f}(\vec{x}(t)) + \kappa B[\vec{x}(t-\tau) - \vec{x}(t)]$$

$$\vec{f}(\vec{x}) = \begin{pmatrix} 10(x_2 - x_1) \\ x_1(28 - x_3) - x_2 \\ x_1 x_2 - 8/3 x_3 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & B_{23} \\ 0 & 0 & 0 \end{pmatrix} \qquad -1 \le B_{23} \le 1$$

$$C_{21} \approx 1.28;\ \mathrm{C}_{23} \approx 1.5 \times 10^{-3};\ D_{2121} \approx 0.16$$

$$D_{2323} \approx 3.79;\ D_{2123} = D_{2321} \approx 9.7 \times 10^{-8};$$

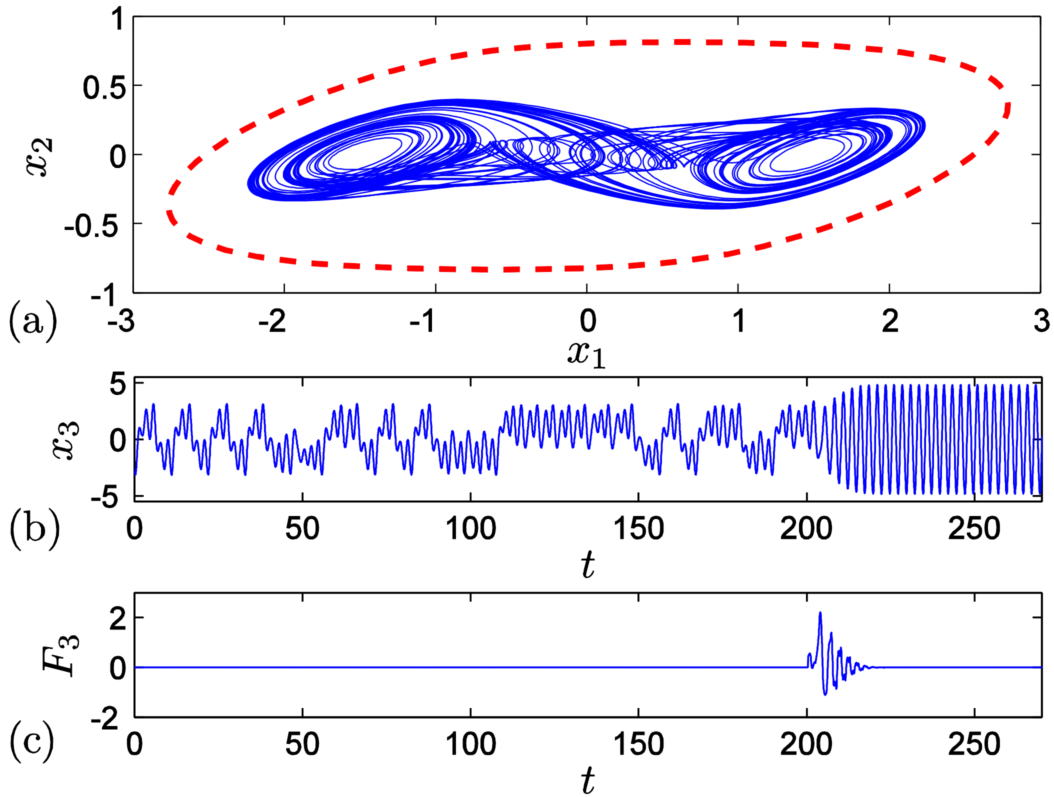
$$|B_{23}| > 0.42;\ \kappa^* \approx 0.78$$

$$B_{23} = 0.5;\ \kappa = 0.865$$

Application to the Chua system

$$\dot{\vec{x}}(t) = \vec{f}(\vec{x}(t)) + \kappa B[\vec{x}(t-\tau) - \vec{x}(t)]$$

$$\vec{f}(\vec{x}) = \begin{pmatrix} 9(x_2 - \phi(x_1)) \\ x_1 - x_2 + x_3 \\ -100/7x_2 \end{pmatrix}$$



$$\phi(x_1) = 2/7x_1 - 3/14(|x_1+1| - |x_1-1|)$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & B_{33} \end{pmatrix} \qquad -1 \le B_{33} \le 1$$

$C_{31} \approx -2.02;\ C_{33} \approx 3.01;\ D_{3131} \approx 2.46$

$D_{3333} \approx 1.85;\ D_{3133} = D_{3331} \approx -2.21;$

$B_{33} = 0.3;\ \kappa^* \approx 0.89$

$\kappa = 1.2$

Conclusions

- A practical recipe of TDFC design for the stabilization of periodic orbits with an odd number of real Floquet multipliers larger than unity is presented
- The algorithm is suited for autonomous systems far from bifurcation points of periodic orbits
- We managed to stabilize the periodic orbits in the Lorenz and Chua systems by the conventional TDFC
- Our findings extend the class of experimental systems where the ETDFC can be implemented.

Details in: K. Pyragas and V. Novicenko, Phys. Rev. E **88**, 012903 (2013)

Acknowledgments

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