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Analytical properties of autonomous systems controlled by extended time-delay feedback in the presence of a small time delay mismatch

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Introduction

The delayed feedback control (DFC) methods are noninvasive, which means that the control signal vanishes if the delay time is adjusted to be equal to the period of a target unstable periodic orbit (UPO). If the delay time differs slightly from the UPO period, a non-vanishing periodic control signal is observed. We derive an analytical expression for this period for a systems controlled by an extended DFC (EDFC) algorithm. Our approach is based on the phase reduction theory adopted to systems with time delay. The key idea is based on splitting the control variable into non-mismatched and mismatched components. The non-mismatched component stabilizes the UPO while the mismatched component induces a small perturbation that can be treated by the phase reduction theory.

Theory

Phase reduction of the oscillator with a small state dependent perturbation:

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}) + \varepsilon \mathbf{P}(\mathbf{X})$$ → Perturbation change period T to Θ.

System demonstrate stable limit cycle behaviour $\mathbf{X}_C(t+T) = \mathbf{X}_C(t)$

From the phase equation we get a perturbed period:

$$\dot{\varphi} = 1 + \varepsilon \mathbf{Z}(\varphi) \mathbf{P}(\mathbf{X}_C(t)) \longrightarrow \Theta = T - \varepsilon \int_0^T \mathbf{Z}(\varphi) \mathbf{P}(\mathbf{X}_C(\varphi)) d\varphi$$

Phase response curve (PRC) of the limit cycle $\mathbf{X}_C(t)$

A 3-D system under time-delayed feedback control:

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}) + \begin{pmatrix} 0 \\ K \\ 0 \end{pmatrix} [x_2(t-\tau) - x_2(t)]$$

control force

The delay time τ differs slightly from the period T of the targeted UPO. The control force can be expanded with respect to mismatch $\tau - T$:

$K[x_2(t-T) - x_2(t)]$ — Non-mismatched component, which stabilize the UPO

$-K(t-T)\dot{x}_2(t-T)$ — Mismatch component, which can be treated by phase reduction method

When the system is under DFC, the profile of the phase response curve $\mathbf{Z}(\varphi)$ is independent on the control gain [1]:

$$\mathbf{Z}(\varphi) = \alpha(K)\rho(\varphi)$$ → PRC of the uncontrolled UPO

coefficient of proportionality

This property allows us to estimate a perturbed period [2]:

$$\Theta(K,\tau) = T + \frac{K}{K-\kappa}(\tau - T) \quad \text{where} \quad \kappa = -\left[\int_0^T \rho_2(\varphi)\dot{x}_{C2}(\varphi)\,d\varphi\right]^{-1}$$

In the EDFC case the control force reads:

$$K\left[\{x_2(t-\tau) - x_2(t)\} + R\{x_2(t-2\tau) - x_2(t-\tau)\} + R^2\{...\} + ...\right]$$

Then the mismatched period is:

$$\Theta(K,\tau) = T + \frac{K}{K-\kappa(1-R)}(\tau - T)$$

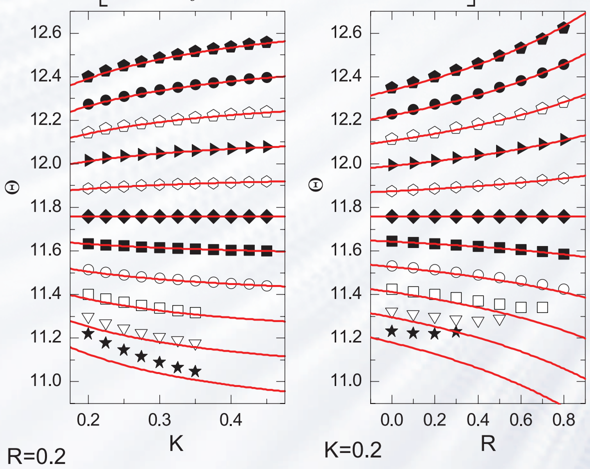
Numerical demonstrations

Rossler system under EDFC:

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 \\ \dot{x}_2 &= x_1 + 0.2x_2 + F(t) \\ \dot{x}_3 &= 0.2 + x_3(x_1 - 5.7) \end{aligned}$$

Here control force:

$$F(t) = K\left[(1-R)\sum_{j=1}^{\infty} R^{j-1} x_2(t-j\tau) - x_2(t)\right]$$



R=0.2 K=0.2

Stabilization of period-two UPO with various mismatches (from bottom to top) $\tau - T$=-1.0; -0.8; -0.6; -0.4; -0.2; 0.0; 0.2; 0.4; 0.6; 0.8; 1.0. Simbols are estimated from the numerical simulations while straight lines represent analytical expression.

Conclusions and references

We have derived an analytical expression which shows in an explicit form how the period of stabilized orbit changes when varying the delay time and the parameters of the control and memory gains.

[1] V. Novičenko and K. Pyragas, Physica D **241**, 1090 (2012).

[2] V. Novičenko and K. Pyragas, Phys. Rev. E **86**, 026204 (2012).

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