Adaptive delayed feedback control algorithm with an iterated feedback gain in the presence of noise

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FIZINIŲ IR TECHNOLOGIJOS MOKSLŲ CENTRAS

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Outline

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- Problem formulation
- Overview
- The idea of algorithm
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- Demonstration for Roessler system
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DFC algorithm:

K. Pyragas, Phys. Lett. A **170**, 421 (1992)

Reviews:

K. Pyragas, Phylos. Trans. R. Soc. London, Ser. A **364**, 2309 (2006);

E. Schoell and **H. G. Schuster**, Handbook of Chaos Control (2008);

P. Hoevel, Control of Complex Nonlinear Systems with Delay (2010) ;

Problem formulation

Our aim is:

To construct an adaptive modification of the DFC method which automatically finds the optimal feedback gain of the DFC systems

We consider the DFC system subjected to a white Gaussian noise so that the variance of delayed difference has a minimum at some optimal value of the feedback gain

Overview

The role of noise in the coupled logistic maps:

D.A. Egolf and J.E.S. Socolar, PRE (1998);
J.E.S. Socolar and D. Gauthier, PRE (1998);
I. Harrington and J.E.S. Socolar, PRE (2004);

Using the speed-gradient method:

J. Lehnert, et al. , Chaos (2011);

Adaptive algorithms for locating USSs and UPOs:

W. Lin, et al., PRE (2010);

Adaptive search for delay time:

V. Pyragas and K. Pyragas PLA (2011);

The idea of the algorithm (I)

Simple example:

$$\dot{x} = -\gamma x + \xi(t)$$

$$\langle \xi(t) \rangle = 0$$

$$\langle \xi(t)\xi(t') \rangle = \varepsilon^2 \delta(t - t')$$

$$\langle x^2(t) \rangle = \langle x_0^2 \rangle e^{-2\gamma t} + \frac{\varepsilon^2}{2\gamma} + \frac{\varepsilon^2}{2\gamma} e^{-2\gamma t}$$

$$\langle x^2(t) \rangle_{t\to\infty} \to \frac{\varepsilon^2}{2\gamma}$$

$$-\gamma = -\frac{\varepsilon^2}{2\langle x^2(t) \rangle_{t\to\infty}}$$

DFC system in presence of noise:

$$\dot{X}(t) = F[X(t), k\Delta s(t)] + \xi(t)$$

$$\Delta s(t) = g[X(t)] - g[X(t-\tau)]$$

$$D^2 = \langle [s(t) - s(t-\tau)]^2 \rangle$$

$$L(k) = -\frac{\varepsilon^2}{2D^2}$$

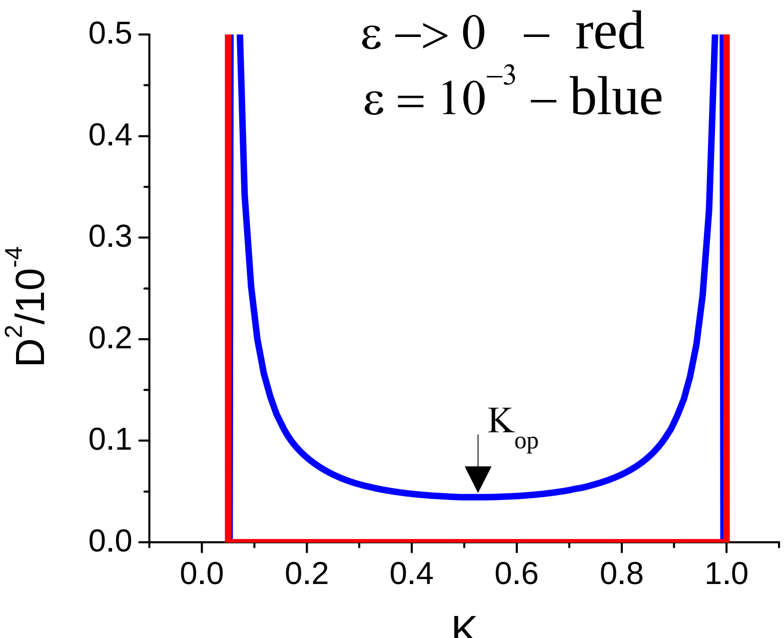
⇒ **Lyapunov exponent** ⇐

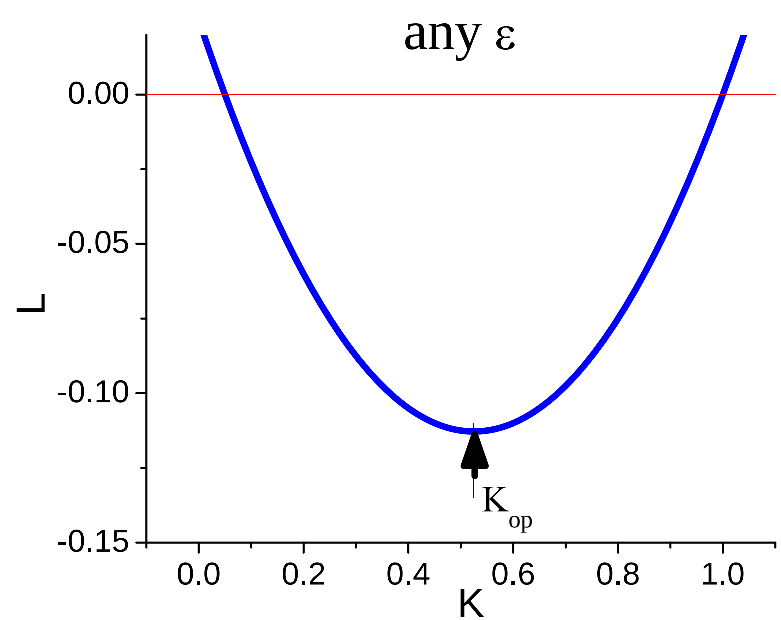
The idea of the algorithm (II)

Example: A noisy linear map under DFC

$$x_{n+1} = \lambda x_n - k(x_n - x_{n-1}) + \xi_n$$

$$D^2 = \left\langle \left(x_n - x_{n-1}\right)^2 \right\rangle \qquad L = -\varepsilon^2 / 2D^2$$





Minimum is flat

Minimum is parabolic

The idea of the algorithm (III)

$$k: \quad \left(k_0, k_0 + \Delta k, \ldots, k_0 + (N-1)\Delta k\right)$$

$$D^2\left(k_j\right) \approx w_j = \frac{1}{\Theta}\int_{t_{on}}^{t_{on}+\Theta} dt\,[s(t) - s(t-\tau)]^2$$

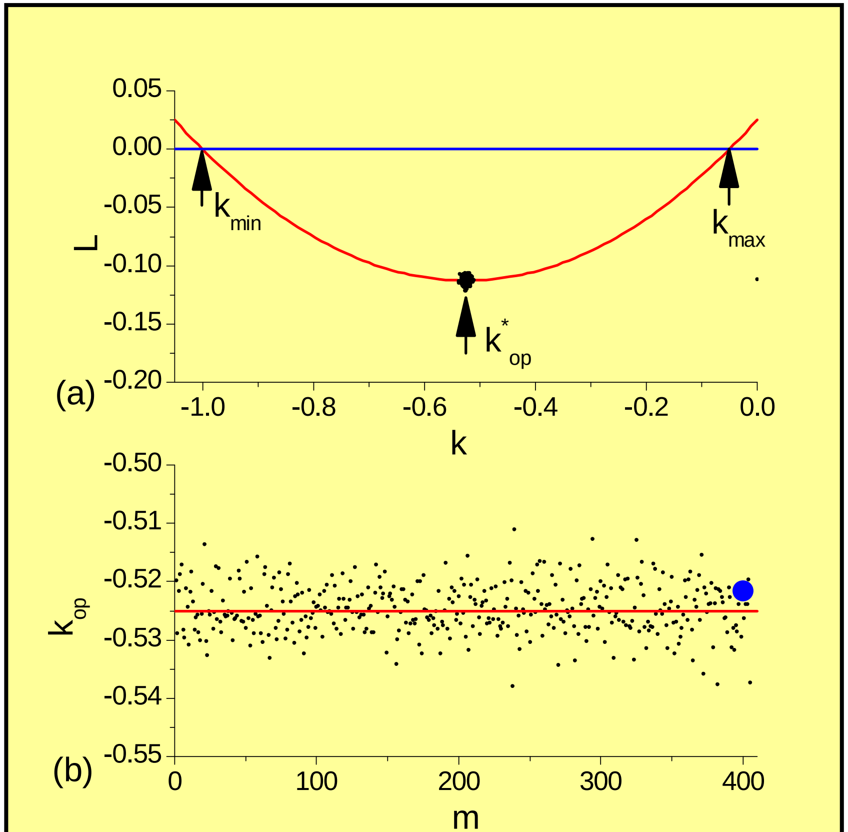
$$L\left(k_j\right) \equiv L_j = -\varepsilon^2 / 2w_j$$

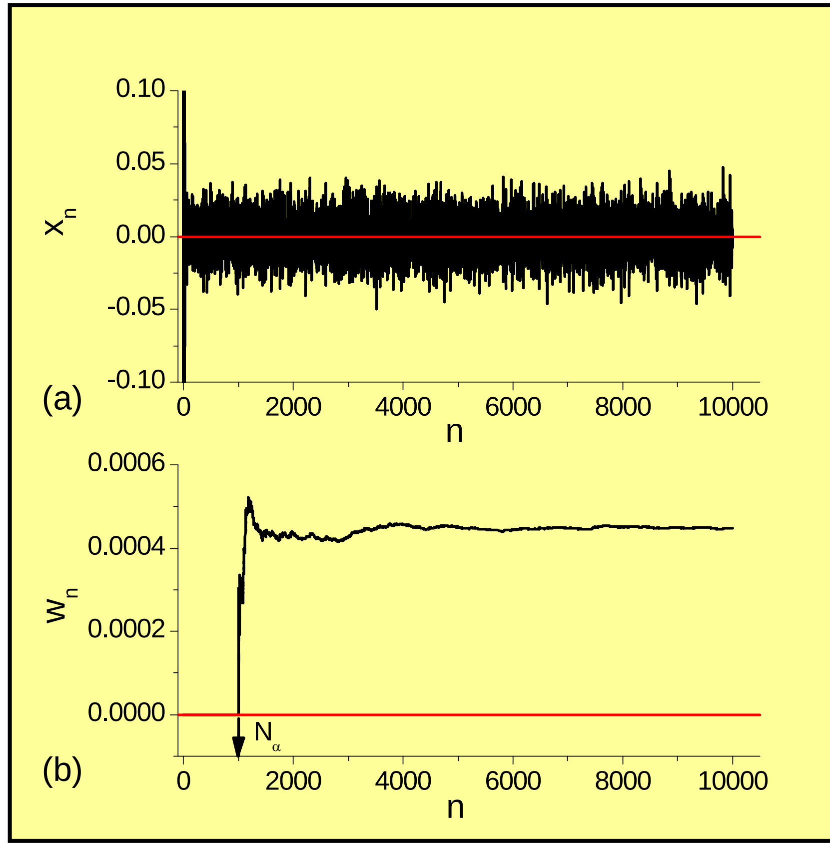
$$L(k) = ak^2 + bk + c$$

$$LSM \rightarrow (a, b, c)$$

$$k_{op} = -b/2a$$

Numerical simulations for the linear map



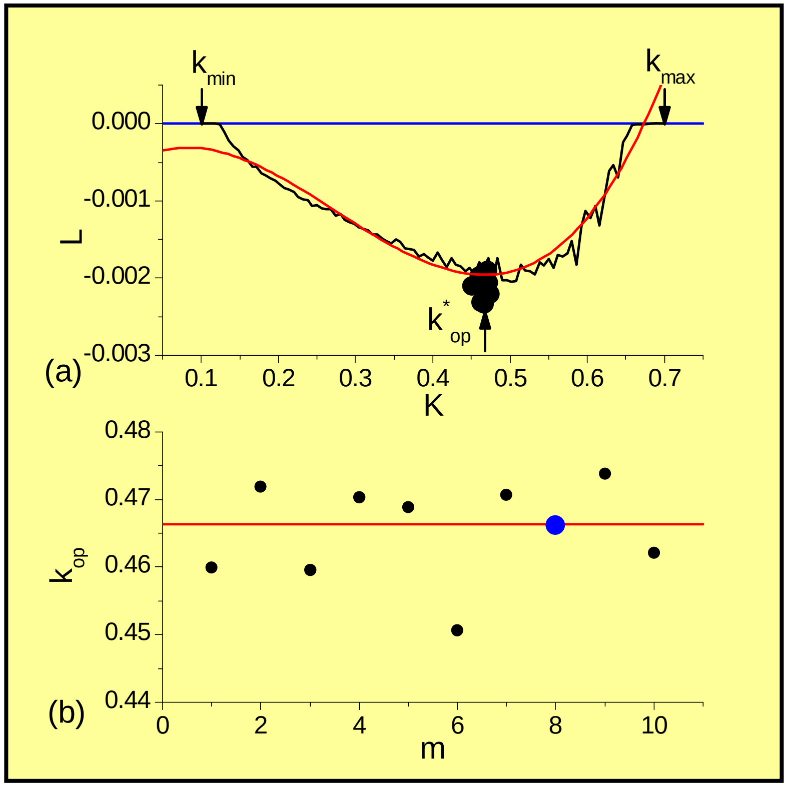


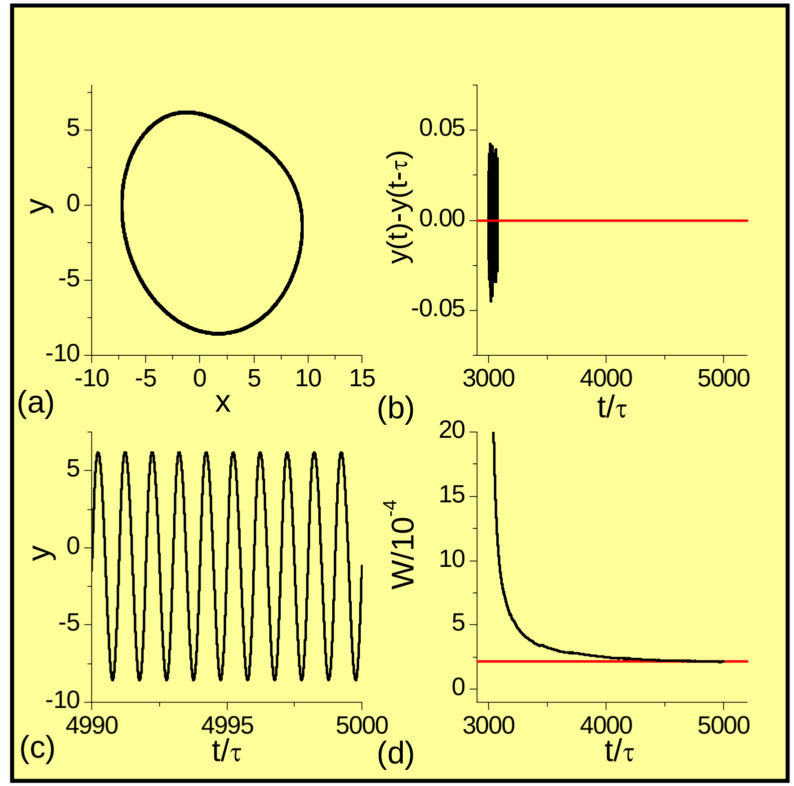
Demonstration for the Roessler system

$$\dot{x} = -y - z + \xi_x(t),$$
$$\dot{y} = x + ay - k[y(t) - y(t-\tau)] + \xi_y(t),$$
$$\dot{z} = b + z(x - c) + \xi_z(t),$$
$$s(t) = y(t),$$
$$\dot{w} = \alpha_{sw}[y(t) - y(t-\tau)]^2.$$

$$\langle \xi_\alpha(t) \rangle = 0, \quad \langle \xi_\alpha(t)\xi_\beta(t') \rangle = \varepsilon^2 \delta_{\alpha\beta}\delta(t - t')$$
$$\alpha, \beta = x, y, z.$$

Numerical simulations for the 1-st UPO





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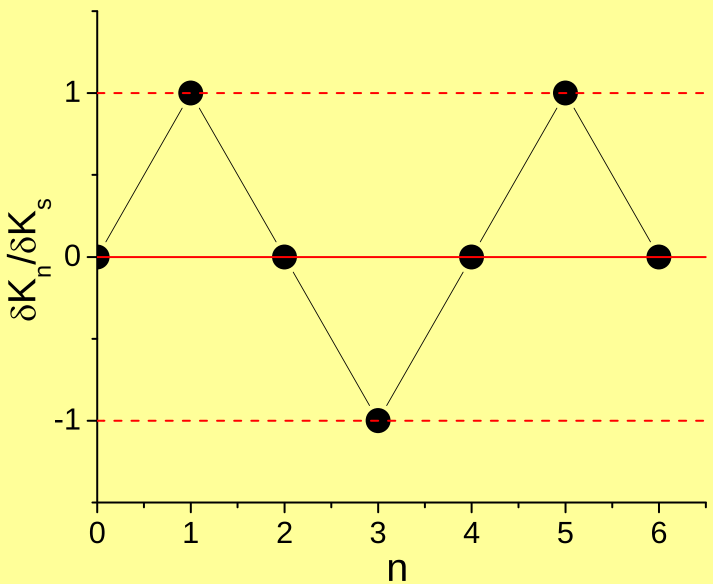
An alternative approach: estimating the sign of gradient of the time-running average



$$x_{n+1} = \lambda x_n + k_n(x_n - y_n) + \xi_{n-1},$$

$$y_{n+1} = x_n,$$

$$\Psi_{n+1} = \nu[\Psi_n + (x_{n+1} - y_{n+1})^2],$$

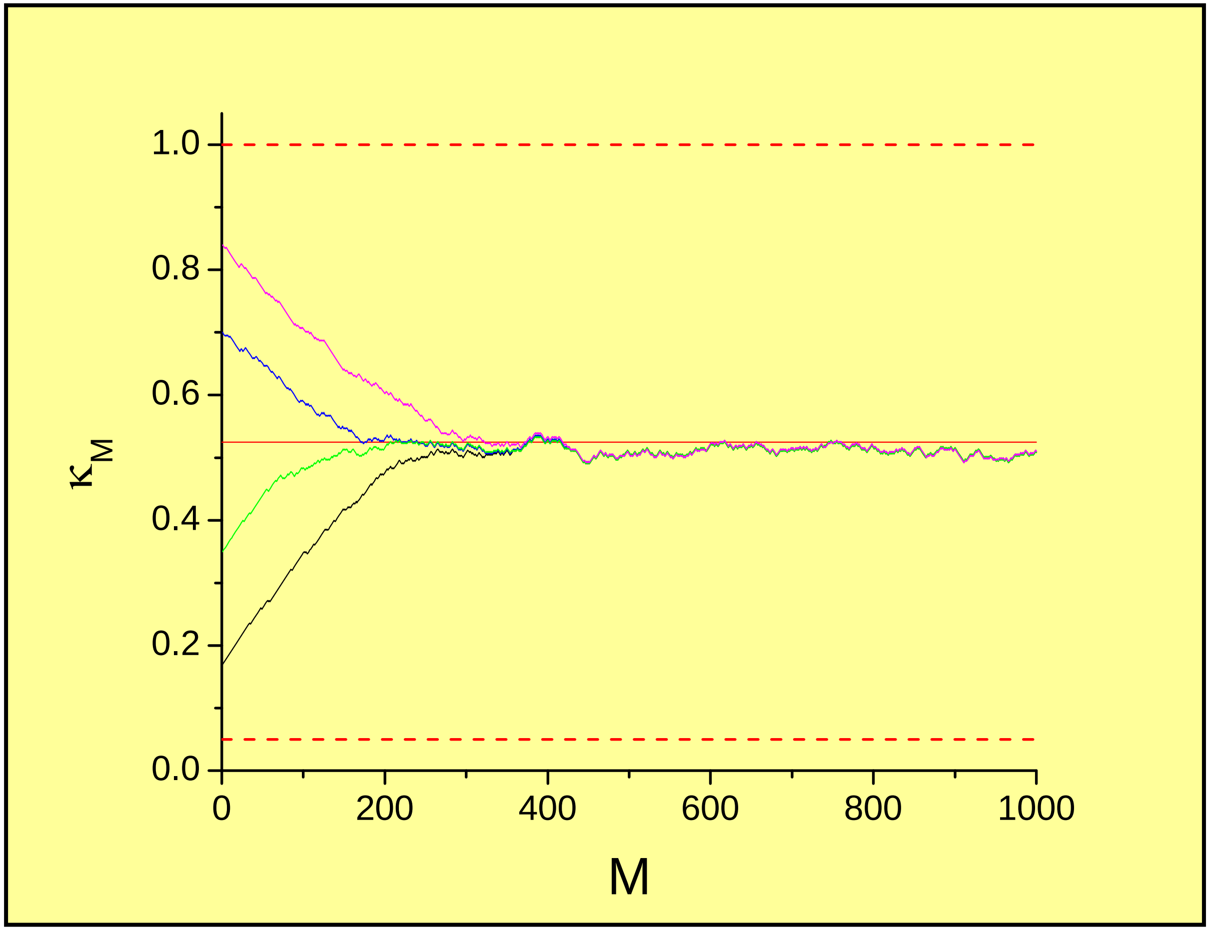
$$S_{n+1} = S_n + (\Psi_n - \Psi_{n-1})\alpha_n,$$

$$k_n = \kappa + \delta k_n,$$

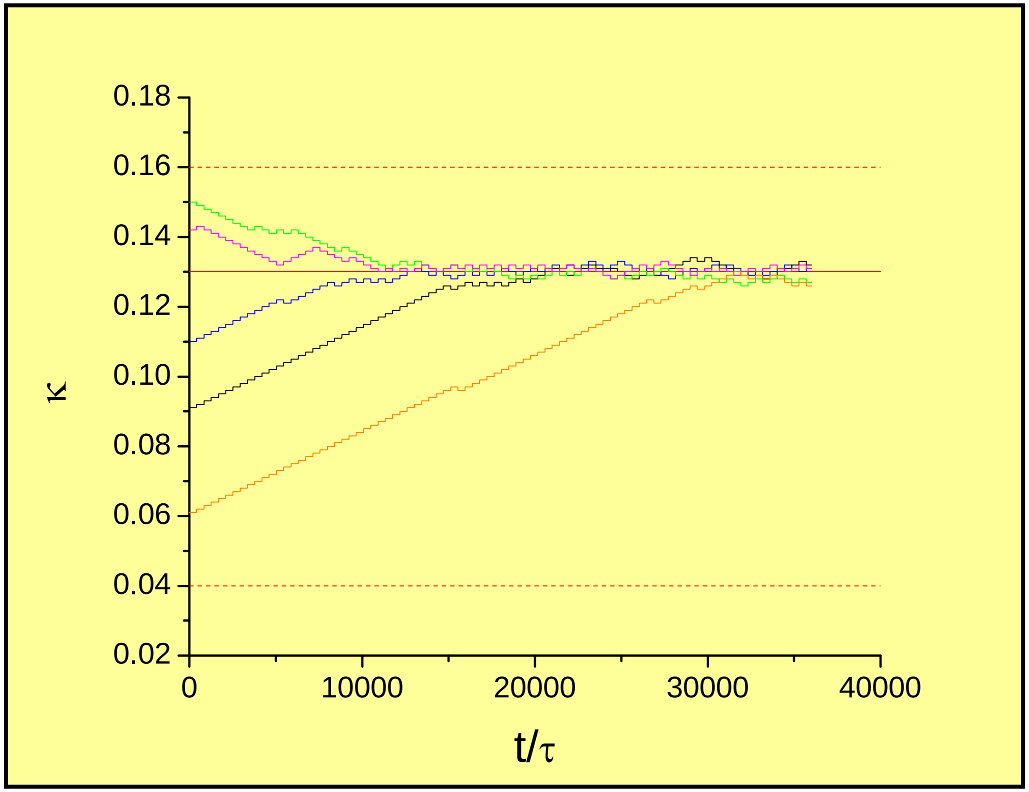
$$\delta k_n = \delta k_s \operatorname{sgn}\left[\sin\left(\frac{\pi}{2} n\right)\right].$$

$$\kappa_{N(M+1)} = \kappa_{NM} - \beta \operatorname{sgn}[S_{N(M+1)}]$$

Numerical simulations for the linear map



Numerical simulations for the normal form of subcritical Hopf bifurcation



Conclusions

- We have proposed an iterative adaptive DFC algorithm that uses the LSM method for the search of the optimal feedback gain.

- The iterated feedback gain converges towards the optimal value and oscillates around it with a small amplitude.

- We also have proposed an adaptive DFC algorithm that estimates the sign of gradient for the variance of the delayed difference in respect to the feedback gain. This sign is used for adjustment of the current feedback gain.

- The both algorithms may be used for stabilization and tracking of experimental systems when the dynamic equations are unknown.

Acknowledgments

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