Twenty years of delayed feedback control technique: Brief review and recent developments

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Outline

- Introduction
- Delayed feedback control (DFC) algorithm
- Experimental implementations
- Linear theory of DFC
- Extended DFC (EDFC)
- Odd number limitation and an unstable controller
- Analytical approaches for DFC systems
- On global properties of the DFC algorithm
- Adaptive modifications
- Phase reduction theory-based treatment of the EDFC
- The list of other DFC problems
- Conclusions

Controlling chaos

E. Ott, C. Grebogy and Y.A. Yorke, PRL 64, 1196 (1990)

3434 citations

Chaotic systems are interesting objects for the control:

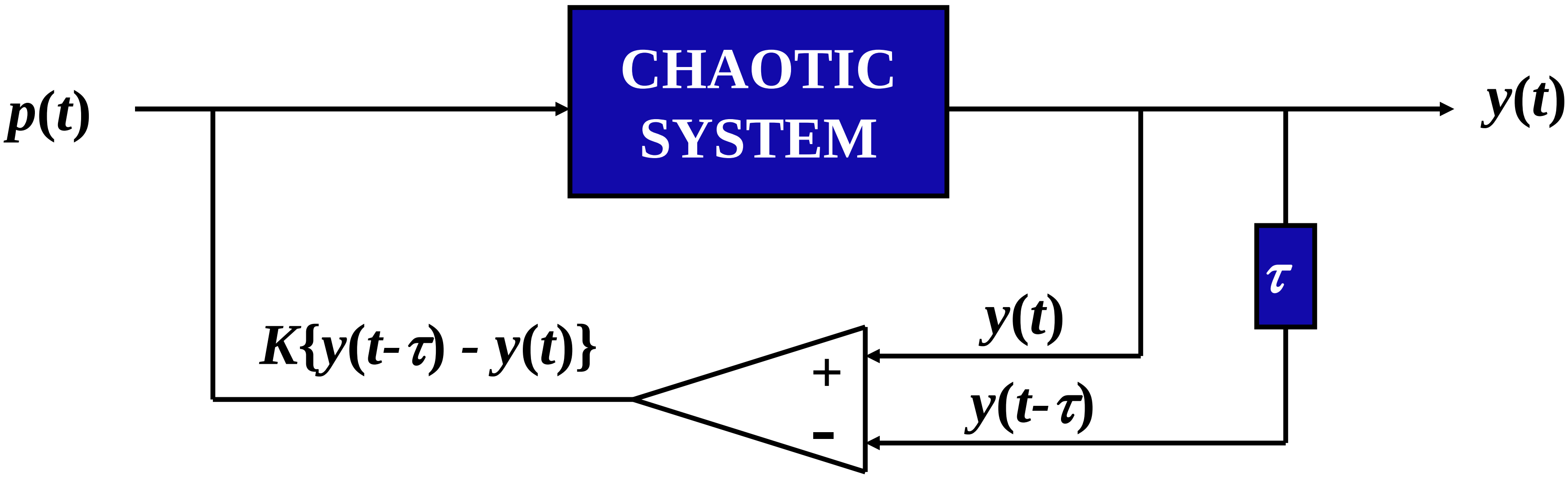
- Sensitive dependence to small perturbations
- An infinite number of unstable periodic orbits (UPOs)
- Stabilization of UPOs with tiny perturbations

OGY algorithm (discrete in time, Poincare section)

Delayed feedback control algorithm

K. Pyragas, Phys. Lett. A 170, 421 (1992)

1672 citations



$$\dot{\vec{x}} = \vec{f}\left(\vec{x}, p\right) \qquad p = p(t) = K\left[y(t) - y(t-\tau)\right]$$

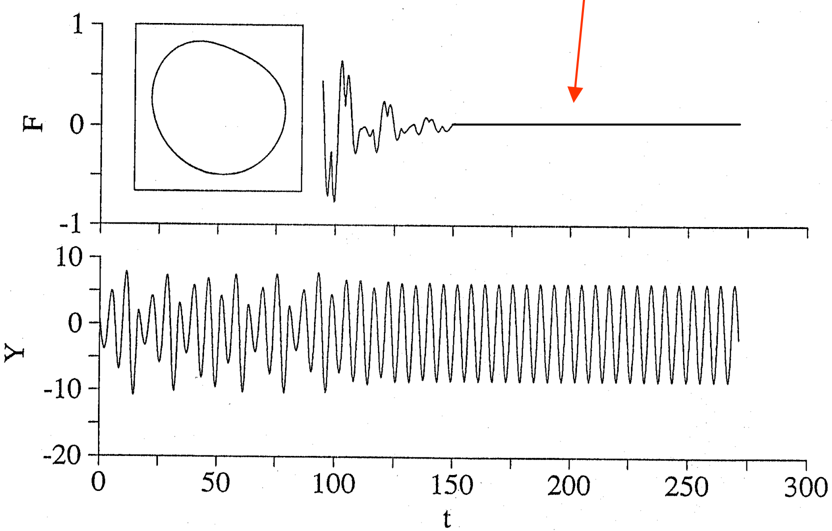
$$y(t) = g(\vec{x})$$

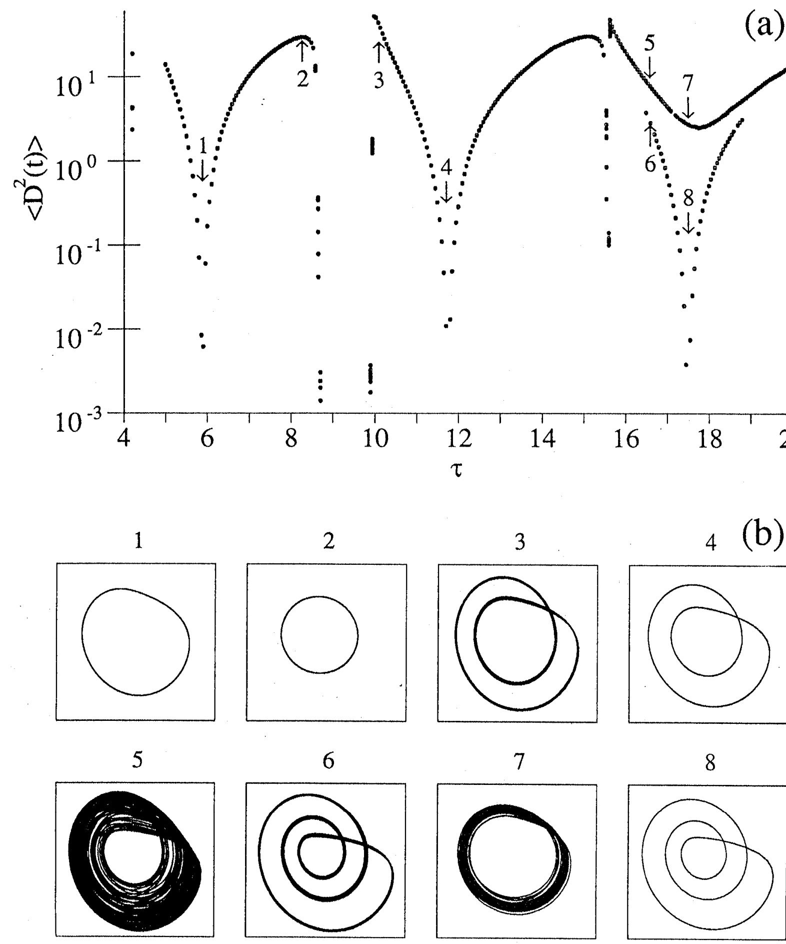
τ – period of unstable periodic orbit

Demonstration for the Roessler system

$$\dot{x} = -y - z$$
$$\dot{y} = x + a\,y \;\; + K\{y(t-\tau) - y(t)\}$$
$$\dot{z} = b + z(x - c)$$

The method is noninvasive





- Matlab animation

Experimental implementations (I)

Electronic chaos oscillators

- Pyragas, Tamaševičius (1993)
- Gauthier et al. (1994)
- Kittel et. al. (1994)
- Celka (1994)
- Socolar et al. (1994)
- Benner et al. (1997)
- Sukow et al. (1997)
- Chang et al. (1998)
- Just et al. (1999, 2000)
- Loewenich et al. (2004)
- Boccaletti et al. (2004)
- Choe et al. (2005)
- Ahlborn, Parlitz (2006)
- Hoehne et al. (2007)
- Tamaševičius et al. (2007)
- Loewenich et al. (2010)

Lasers

- Belawski et al. (1994)
- Erneux et al. (1995)
- Lu, Yu, Harrison (1998)
- Arecchi et al. (2002)
- Boccaletti et al. (2004)
- Bielawski et al. (2005)
- Schikora et al. (2006, 2008)
- Dahms et al. (2010)
- Schicora at al. (2011)

Chemical systems

- Schneider et al. (1993)
- Parmananda et al. (1999)
- Tsui, Jones (2000)
- Kiss et al. (2000)
- Kim et al. (2001)
- Bertram et al. (2003)
- Beta et al. (2003)
- Kiss et al. (2006, 2008)

Experimental implementations (II)

Mechanical pendulums

- **Hikihara, Kawagoshi (1996)**
- **Christini et al. (1997)**
- **Sieber et al. (2008)**

Gas discharge systems

- **Pierre et al. (1996)**
- **Mausbach et al. (1997)**
- **Wei et al. (2004)**

Plasma

- **Gravier et al. (1999)**
- **Fukuyama et al. (2002)**
- **Fukuyama et al. (2006)**

Ferromagnetic resonance

- **Ye et al. (1995)**
- **Benner et al. (1997)**

DC-DC boost converter

- **Natsheh et al. (2009)**

Walking control of robots

- **Sugimoto, Osuka (2002)**

Cardiac systems

- **Hall et al. (1997)**
- **Rappel et al. (1999)**
- **Berger et al. (2007)**

Chaotic Tailor-Couette flow

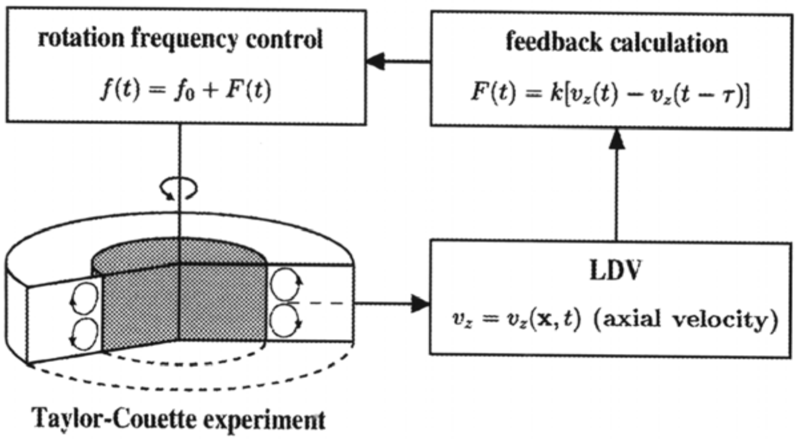
- **Lüthje et al. (2001)**

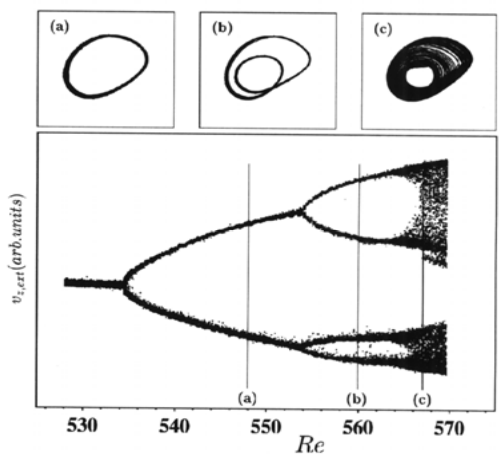
Atomic force microscope

- **Yamasue et al. (2009)**

Control of chaotic Taylor-Couette flow

Lüthje, Wolf, Pfister, PRL (2001)

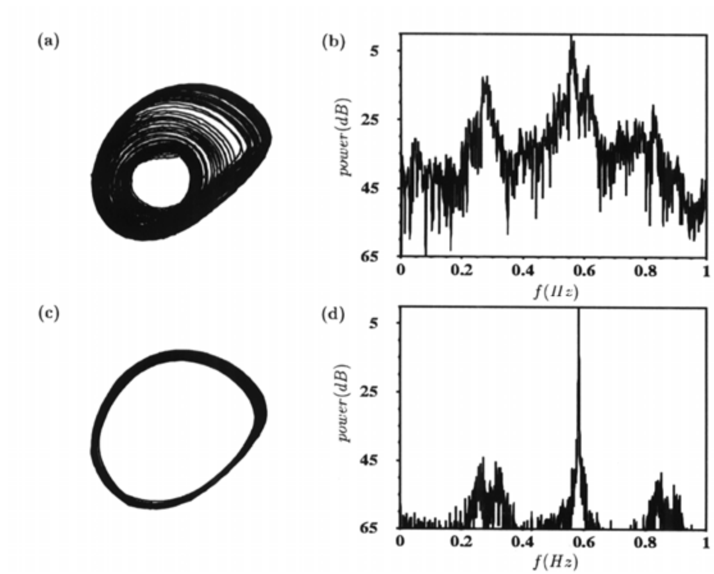




$$\mathrm{Re} = 2\pi f\,(r_o - r_i)\,r_i/\nu$$

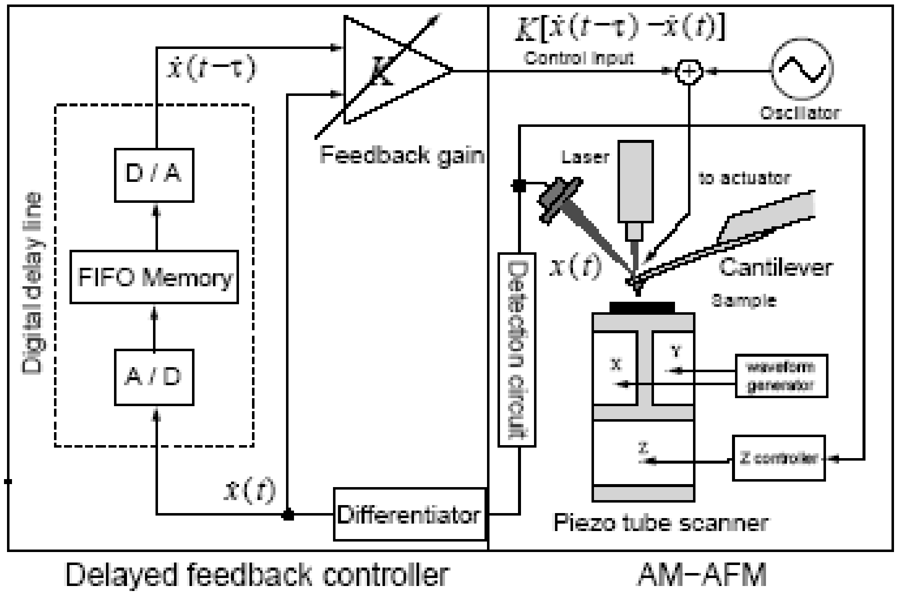
$v_z(\mathbf{x},t)$ - output (axial velocity)

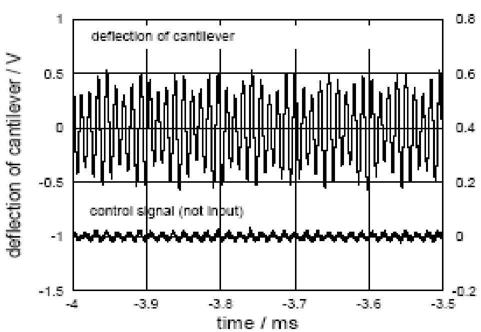
f - control parameter



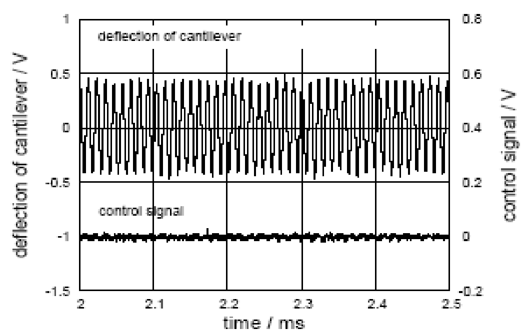
Controlling chaos in dynamic-mode atomic force microscope

K.Yamasue et al., PLA (2009)





without control



with control

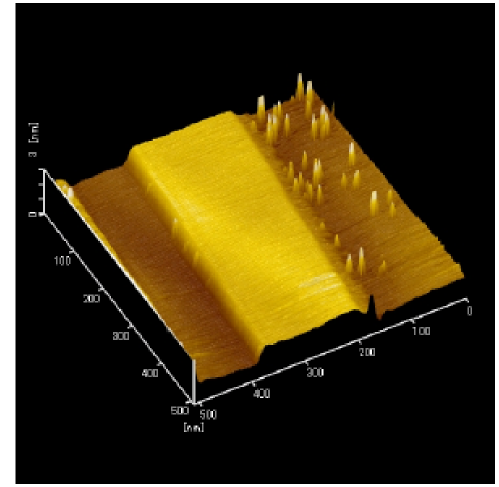


Image without control

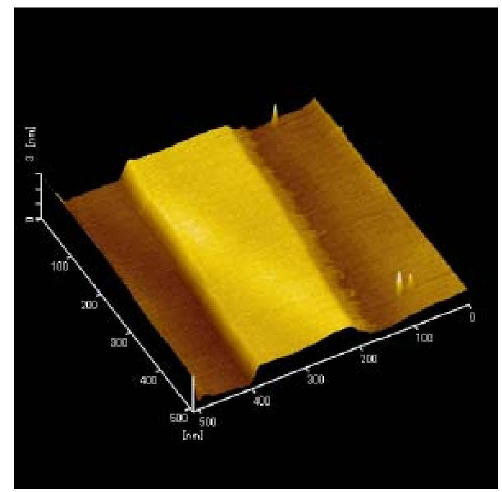
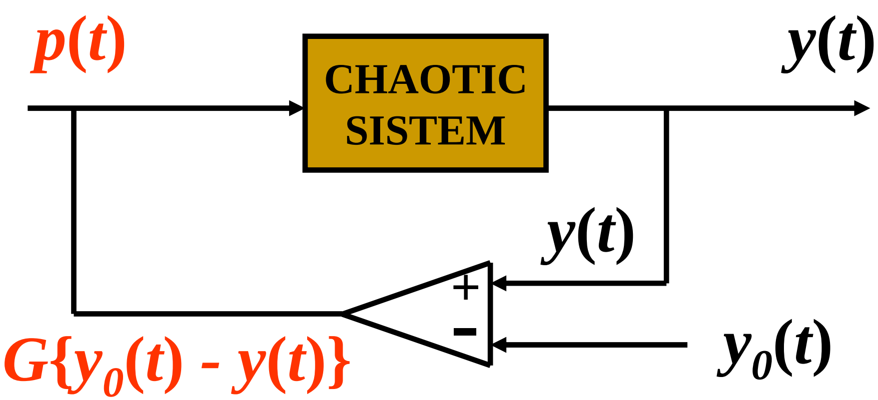


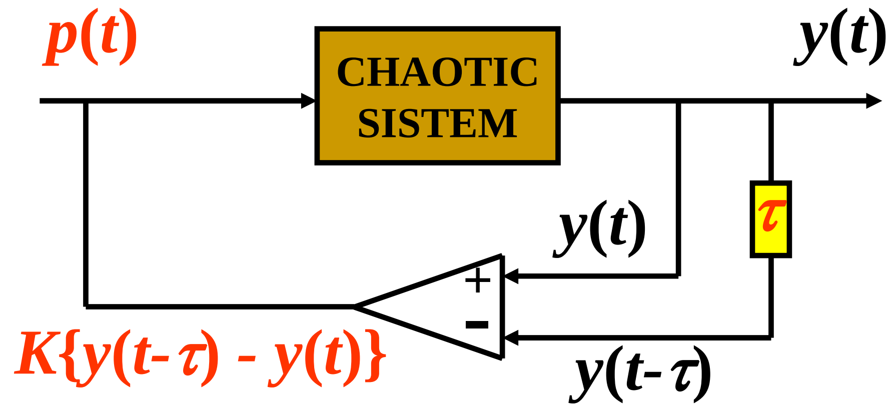
Image with control

Proportional feedback versus delayed feedback

Proportional feedback



Delayed feedback



system:	output:	UPO at $p = 0$:
$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x}, p)$	$y = g(\vec{x})$	$\vec{x}_0(t+\tau) = \vec{x}_0(t)$

$$y_0(t) = g(\vec{x}_0(t))$$

$$p = G\{y_0(t) - y(t)\}$$

$$p = K\{y(t-\tau) - y(t)\}$$

Floquet Exponents

Proportional feedback **Delayed feedback**

$$\delta\vec{x} = \vec{x} - \vec{x}_0$$

$$\delta\dot{\vec{x}} = [A(t) + GB(t)]\delta\vec{x}$$

$$\delta\dot{\vec{x}} = A(t)\delta\vec{x} + KB(t)[\delta\vec{x}(t-T) - \delta\vec{x}]$$

$$A(t+T) = A(t), \quad B(t+T) = B(t)$$

$$\boxed{\delta\vec{x} = e^{\Lambda t}\vec{u}(t), \quad \vec{u}(t+T) = \vec{u}(t)} \Rightarrow \boxed{\delta\vec{x}(t-T) = e^{-\Lambda T}\delta\vec{x}(t)}$$

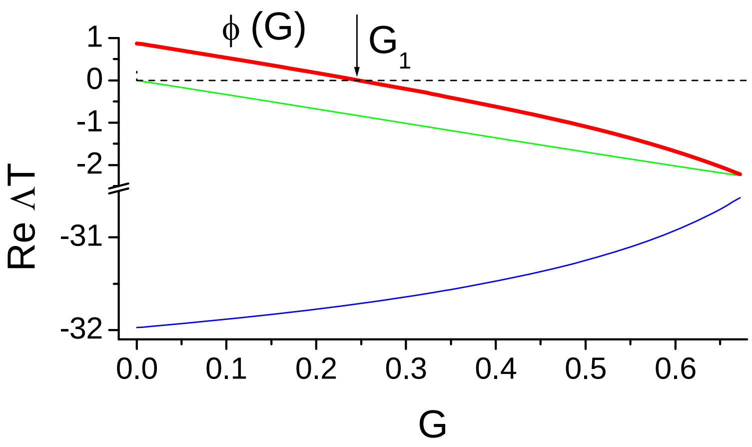
$$\delta\dot{\vec{x}} = \left\{A(t) + K\left[1 - e^{-\Lambda T}\right]B(t)\right\}\delta\vec{x}$$

$$\boxed{\Lambda_j = F_j(G)}$$

$$\boxed{G \Rightarrow K\left[1 - e^{-\Lambda T}\right]} \Rightarrow \boxed{\Lambda = F_j\left(K\left[1 - e^{-\Lambda T}\right]\right)}$$

Floquet Exponents

Proportional feedback



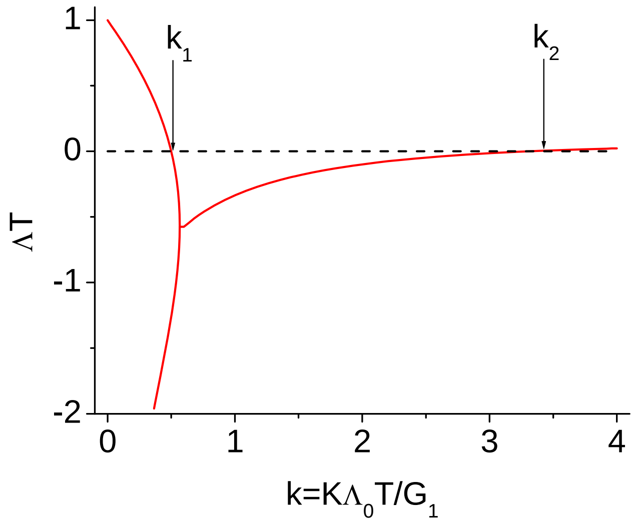
$$\Lambda T = F(G)T = \phi(G) + i\pi$$

Linear approximation (W. Just):

$$\phi(G) = \Lambda_0 T(1 - G/G_1)$$

Λ_0 **- real part of the Floquet expo-nenet of the control free UPO**

Delayed feedback



$$G = K\left[1 - e^{-\Lambda T}\right] \qquad K_1 = G_1/2$$

$$\Lambda T = \Lambda_0 T(1 - G/G_1) + i\pi$$

Stabilisation is possible only if $\boxed{\Lambda_0 T \le 2}$

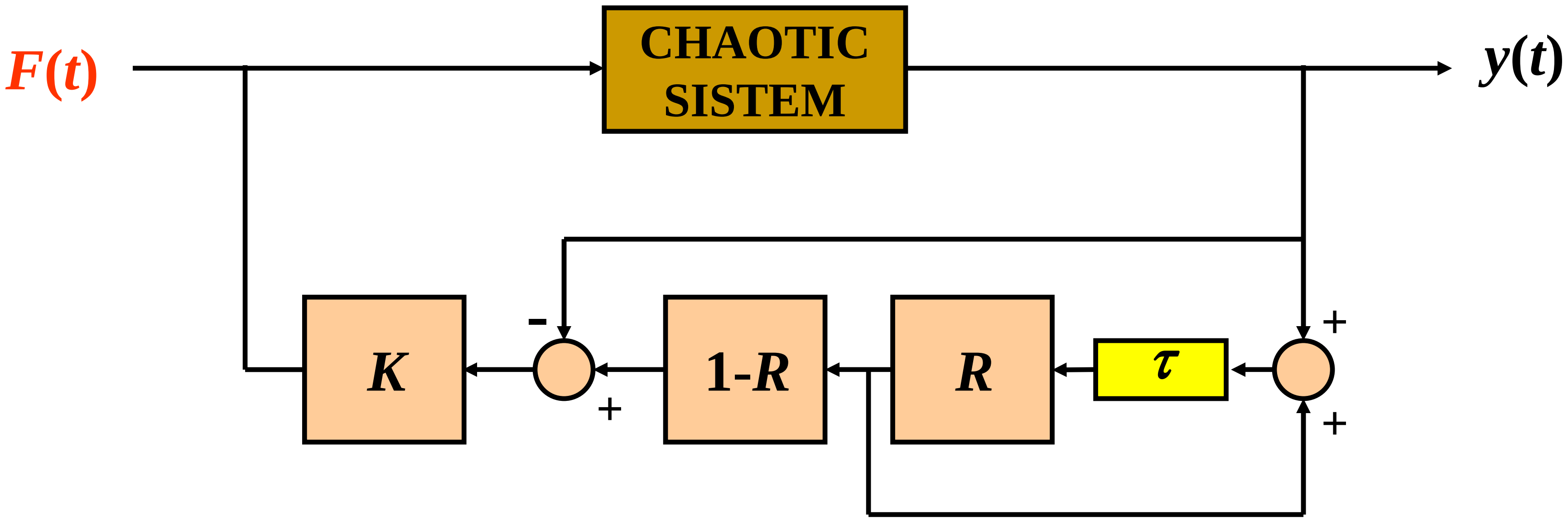
[Non-linear dependence $\phi = \phi$ (G) is considered in: K. Pyragas, Phys. Rev. E 66, 026702 (2002)]

Important modification: Extended DFC (EDFC)

Socolar, Sukov, Gauthier, PRE (1994) **[Pyragas, PLA (1995)]**

$$F(t) = y(t) - y(t-\tau) + R[y(t-\tau) - y(t-2\tau)] + R^2[y(t-2\tau) - y(t-3\tau)] + \cdots$$
$$= y(t) - (1-R)\sum_{k=1}^{\infty} R^{k-1} y(t-k\tau),$$

$|R|<1$ - convergence condition



- **Enables the stabilization of highly unstable orbits**
- **Simpe implementation with a single delay line (Fabri-Perrot inerfer.)**

Advantages of the EDFC

Approximate theory

$$G = K\frac{1-e^{-\Lambda T}}{1-e^{-\Lambda T}R}$$

$$\Lambda T = \Lambda_0 T\left(1 - G/G_1\right) + i\pi$$

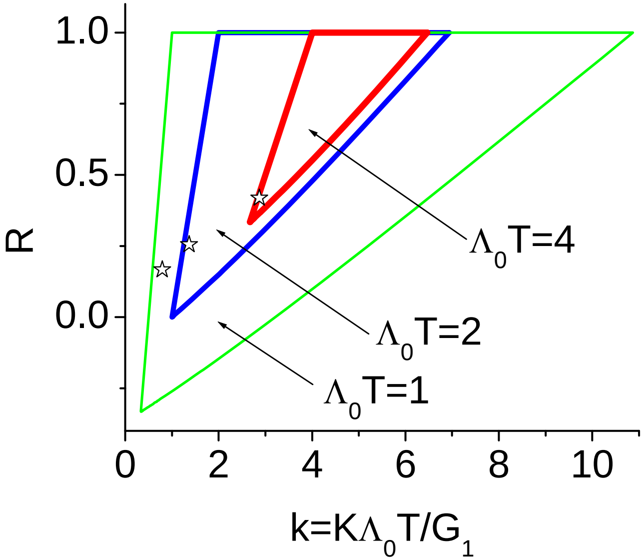
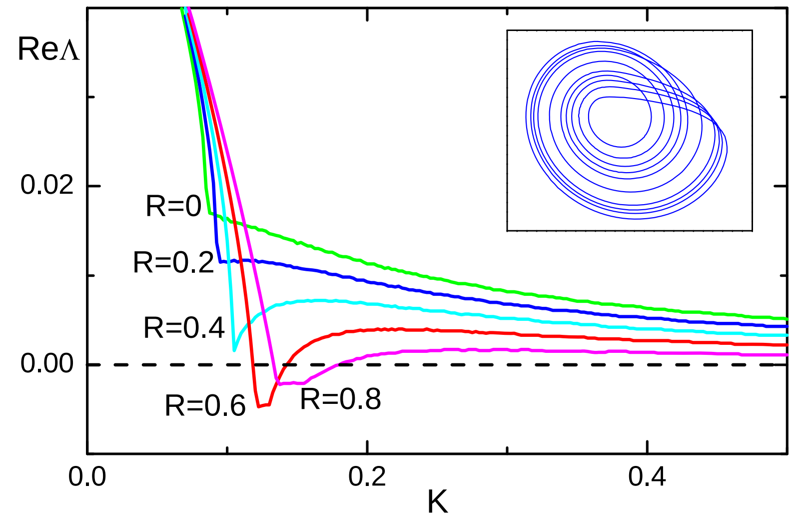


Illustration for the Rössler system

$$\dot{x} = -y - z$$
$$\dot{y} = x + ay \; - KF(t)$$
$$\dot{z} = b + z\left(x - c\right)$$

$$F(t) = y(t) - y(t-T) + R\,F(t-T)$$

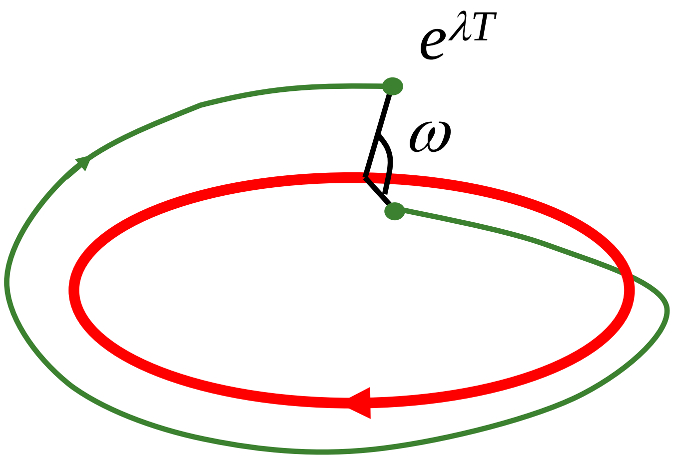


Odd number limitation (ONL)

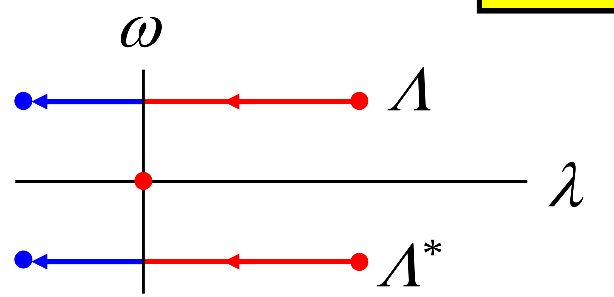
Ushio (1996), Nakajima and Ueda (1997,1998), Just at al. (1997)

Limitation: The DFC method fails for any periodic orbits with an odd number of real Floquet multipliers >1

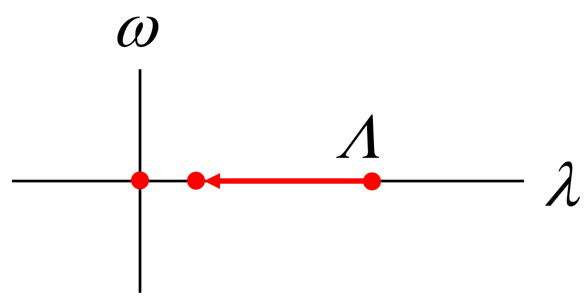
FE: $\Lambda = \lambda + i\omega$ **FM:** $\mu = e^{\Lambda T}$



The method works for $\omega \neq 0$



The method fails for $\omega = 0$



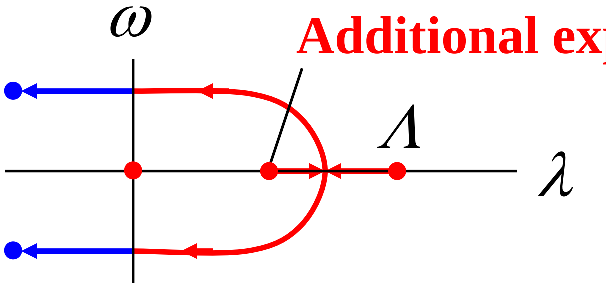
B. Fiedler at al., PRL (2007) → **Refuting for autonomous systems**

Unstable delayed feedback controller

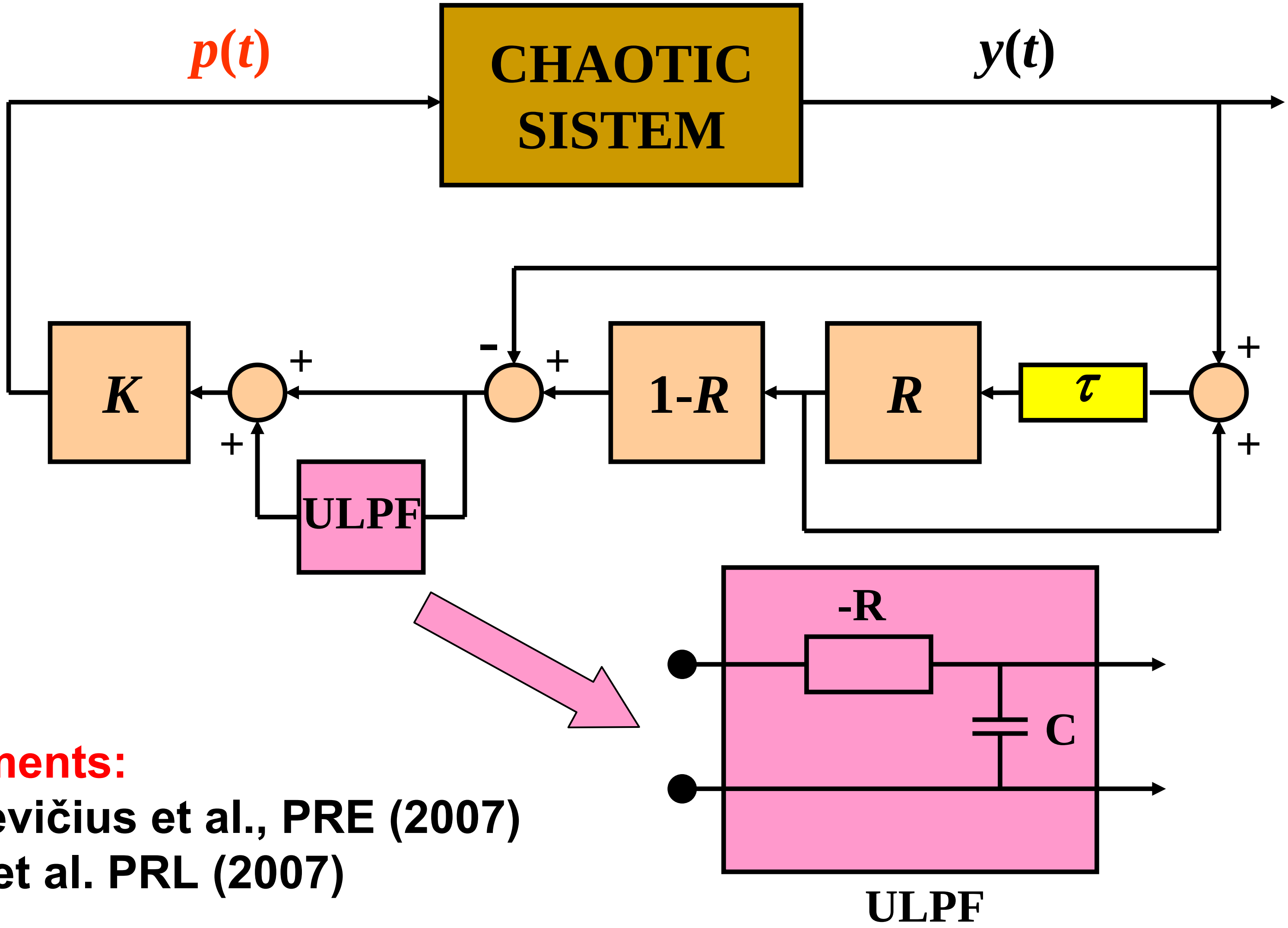
K. Pyragas, Phys. Rev. Lett. 86, 2265 (2001)

Counterintuitive idea:

Artificially enlarge a set of real Floquet multipliers greater than unity to an even number by introducing into a feedback loop an unstable degree of freedom



Block diagram of the unstable EDFC (UEDFC)



Experiments:

Tamaševičius et al., PRE (2007)

Hoene et al. PRL (2007)

Demonstration for the Lorenz system

$$\dot{x} = -\sigma(y+z)$$
$$\dot{y} = rx - y - xz - KF_u(t)$$
$$\dot{z} = xy - bz$$

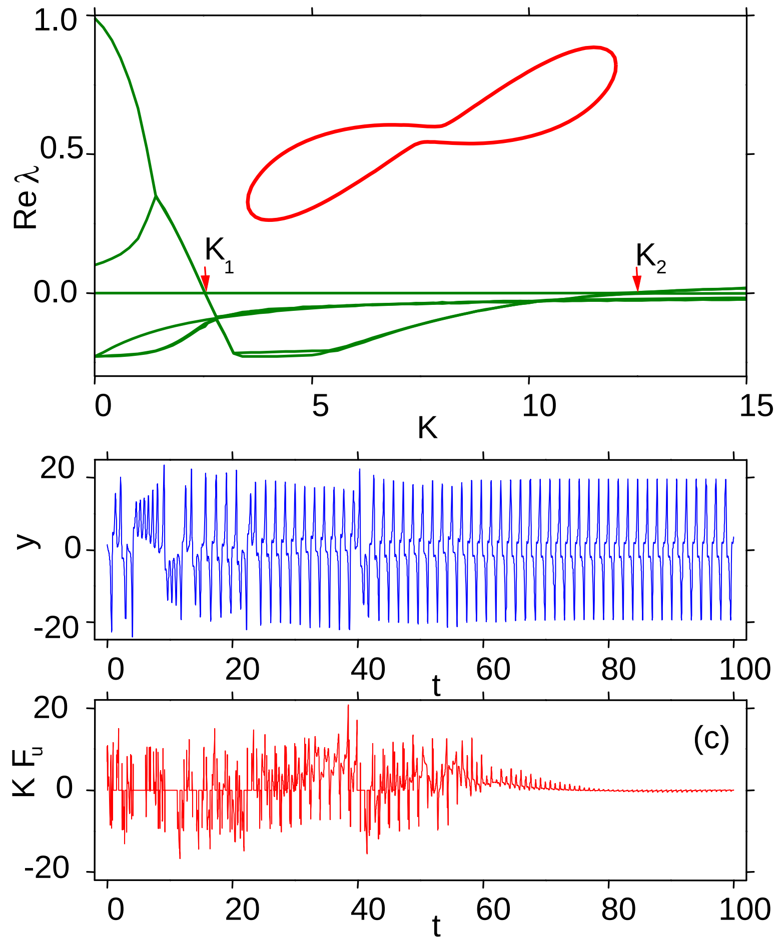
UEDFC:

$$F_u(t) = F(t) + w(t)$$

$$F(t) = y(t) - y(t-\tau) + R\,F(t-\tau)$$

$$\dot{w} = \lambda_c^0 w(t) + \left(\lambda_c^0 - \lambda_c^\infty\right) F(t)$$

$$R < 1, \quad \lambda_c^0 > 0$$



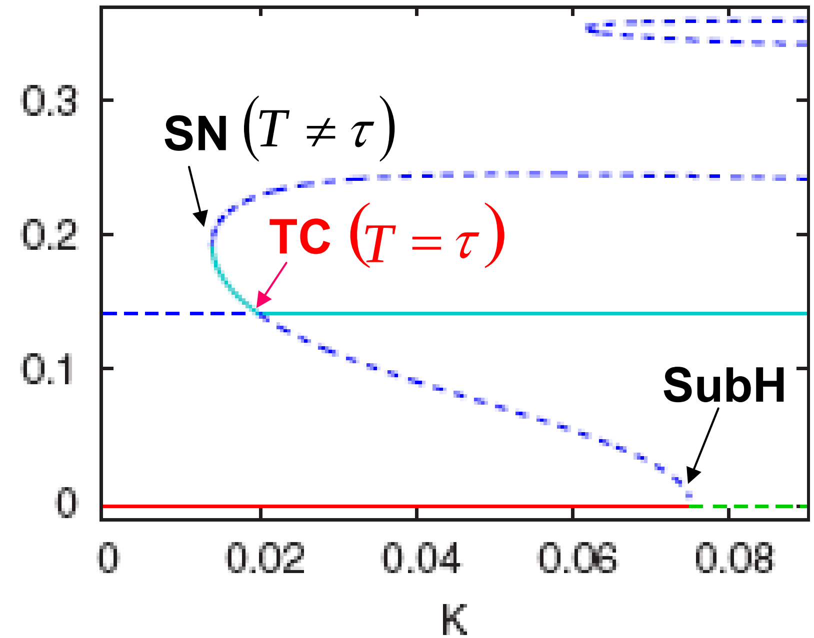
The odd number limitation revisited (I)

Fiedler at al., PRL (2007) → **Counterexample**

Normal form of SubH bifurcation under DFC:

$$\dot{z}(t) = \left[(\lambda + i) + (1 + i\gamma)|z(t)|^2\right]z(t) - Ke^{-i\beta}\left[z(t) - z(t-\tau)\right]$$

Complex gain (rotaiting feedback)



- **DFC induces a pair of orbits (a saddle-node bifurcation) with $T \neq \tau$**

- **The UPO coalisce with iduced orbit at a transcritical bifurcation and exchange the stability**

The odd number limitation revisited (II)

Further publications:

Just at al., PRE (2007)
Flunkert, Schoell (2011)
Brown et al., Physica D (2011)
Schicora at al., PRE (2011) → **Laser experiment**

Despite the refuting of the odd number theorem, the odd number problem still exists:

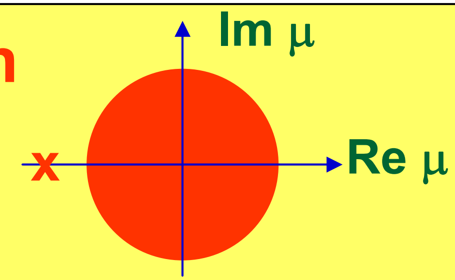
- The odd number theorem remains true for nonautonomous systems
- In practice, the DFC method does not work for many autonomous systems with an odd number of real Floquet multipliers
- It seems that the Feedler's results can be applied for practical stabilization of UPOs only when they are close to a subcritical Hopf bifurcation
Far away from the SubH bifurcation the method based on an unstable controller seems to be more efficient

Analytical approaches for DFC systems (linear analysis)

- **Close to a period-doubling bifurcation**

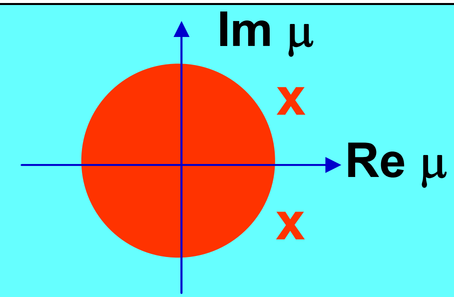
W. Just at al, Phys. Rev. Lett. (1997, 2004)

K. Pyragas, Phys. Rev. E (2002)



- **Close to a Nejmark-Sacker bifurcation**

T. Pyragienė, K. Pyragas, Phys. Rev. E (2005)

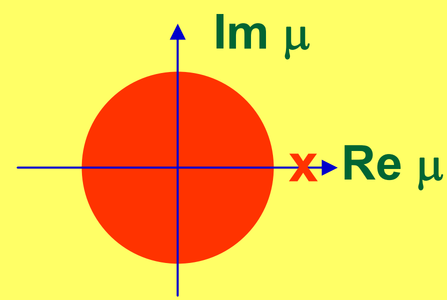


- **Close to a subcritical Hopf bifurcation based on unstable controller**

K. Pyragas, at al, Phys. Rev. E (2004)

V. Pyragas, K. Pyragas, Phys. Rev. E (2006)

K. Hoene et al., Phys. Rev. Lett. (2007)



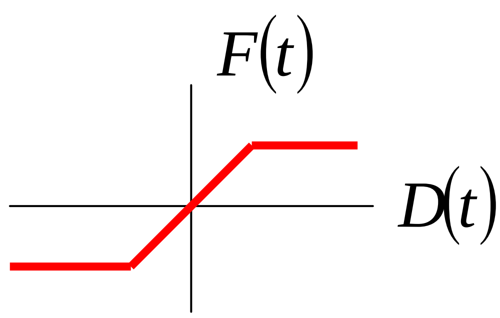
On global properties (basins of attraction) of the DFC

- The linear analysis is insufficient to guarantee experimental success of DFC algorithm, because the control performance may strongly depend on the basin of attraction of the stabilized state.

- To improve the global properties of the DFC algorithm a simple algorithm based on the limitation of the size of the control force by a simple cutoff is usually used [Pyragas, PLA (1992)]:

$$D(t) = k[y(t) - y(t-\tau)]$$

$$F(t) = \begin{cases} D(t), \ for\ |D(t)| \le \varepsilon \\ \varepsilon, \ for\ |D(t)| > \varepsilon \end{cases}$$



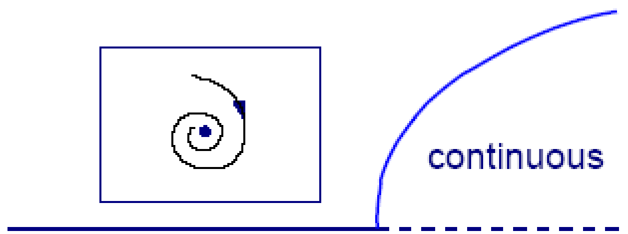
Unfortunately, this approach is not always successful

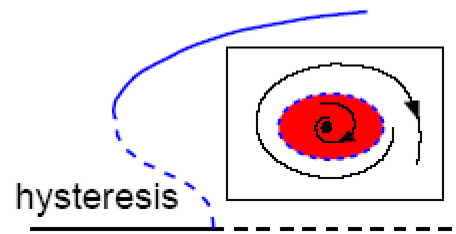
Estimation of basins of attraction based on bifurcation

Just et al., Physica D (2004); Loewenich et al., PRL (2004); Hoene et al., PRL (2007); Erzgräber and Just, Physica D (2009)

The main idea:

Define the type of bifurcation on the control boundaries





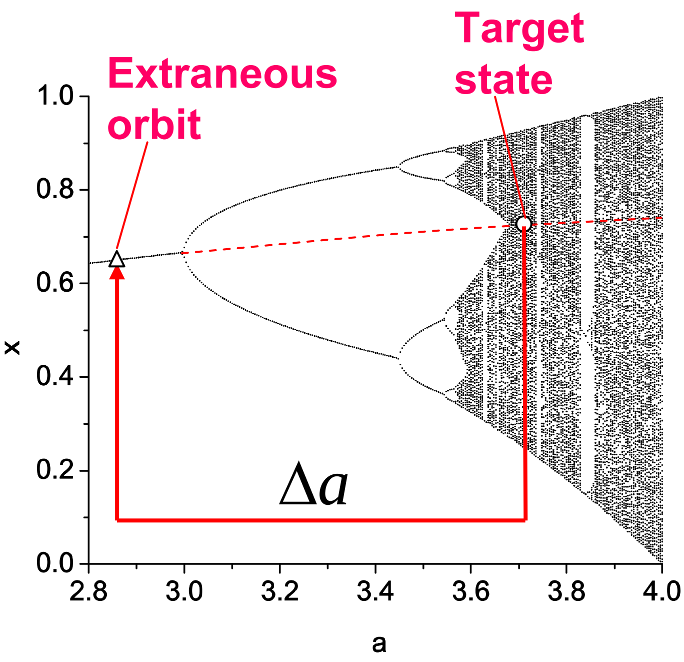
Continuous (supercritical) bifurcation	Discontinuous (subcritical) bifurcation
↓	↓
Large basin of attraction	Small basin of attraction

Increasing the basin of attraction via a two-step algorithm (I)

Tamaševičius et al., PRE (2007)

Simple example logistic map:

$$x_{n+1} = a x_n (1 - x_n)$$



Control algorithm

$$x_{n+1} = [a - (1 - C)\Delta a + Ck(x_n - y_n)] x_n (1 - x_n)$$
$$y_{n+1} = x_n$$

Free system: $C = 1, k = 0$

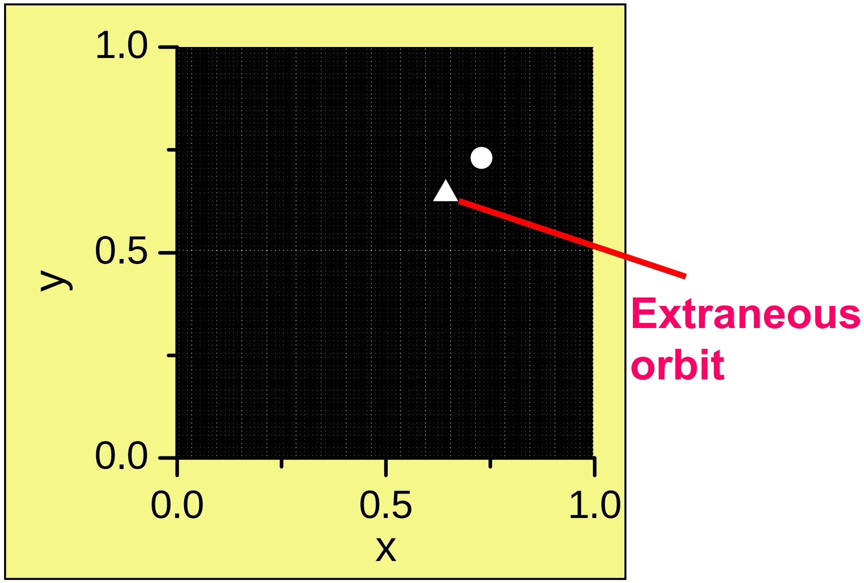
First step: $C = 0, k = 0$

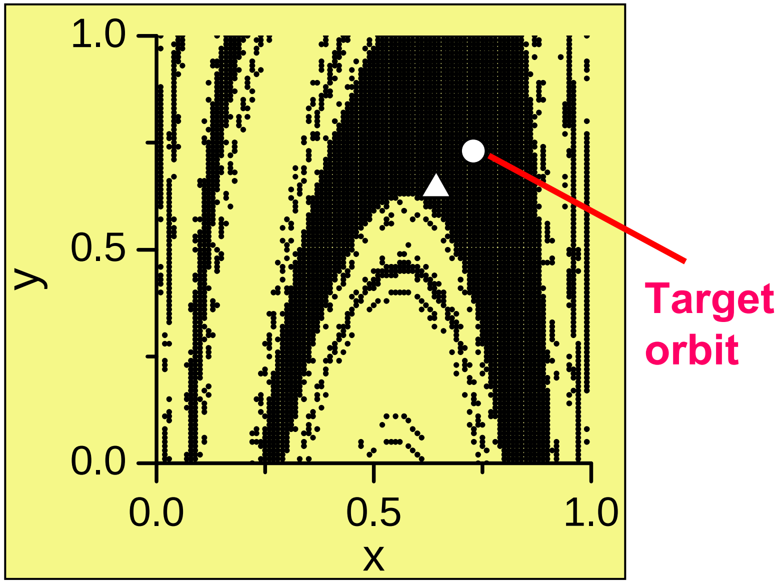
Second step: $C = 0, k \neq 0$

Stability condition of the target orbit

$$\frac{a-3}{2}\frac{a^2}{a-1} < k < \frac{a^2}{a-1}$$

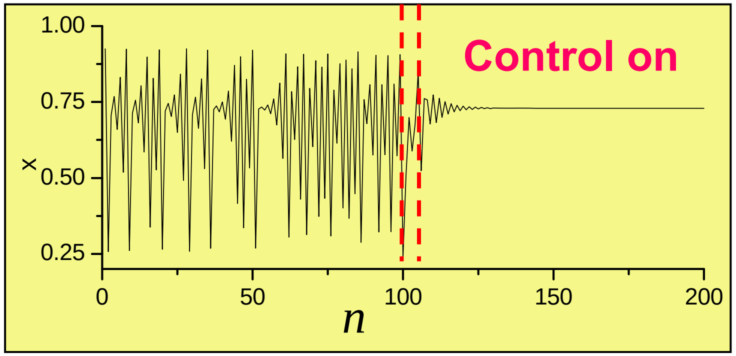
Increasing the basin of attraction via a two-step algorithm (II)





Basin of attraction of the extraneous orbit (*C*=0)

Basin of attraction of the target orbit (*C*=1, *k*=2)



Two-step algorithm essentially enlarges the basin of attraction of the target orbit !!!

Using ergodicity to enlarge the basin of attraction

Pyragas et al., PRE (2009)

- Ergodicity: any chaotic system visits a neighborhood of each periodic orbit with the finite probability

- The idea of using ergodicity in chaos control research has been first formulated in the seminal paper by Ott, Grebogy and Yorke in 1990 and has been employed in their OGY control algorithm

Main idea:

Do not perturb the system until it is far away from the desired UPO and activate the control when it approaches the UPO

The peculiarity of the DFC systems:

The closeness of the system to the UPO has to be considered in infinite dimensional phase space

$$\tau_f \dot{w} = |y(t) - y(t-\tau)| - w$$

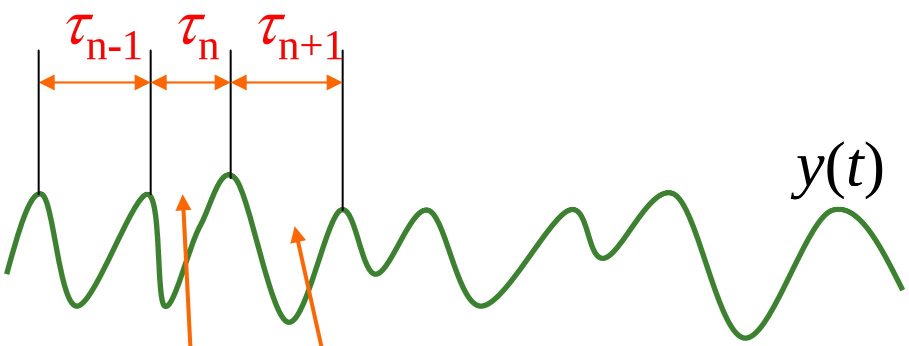
$$w < \varepsilon$$

The running closeness can be estimated by LPF (follows from phase space reconstruction by using the delay coordinates)

Adaptive DFC (discrete version)

Kittel, Parisi, Pyragas, PLA (1995)

An automatic adjustment of the delay time is important for autonomous system, because the period of UPO is unknown beforehand



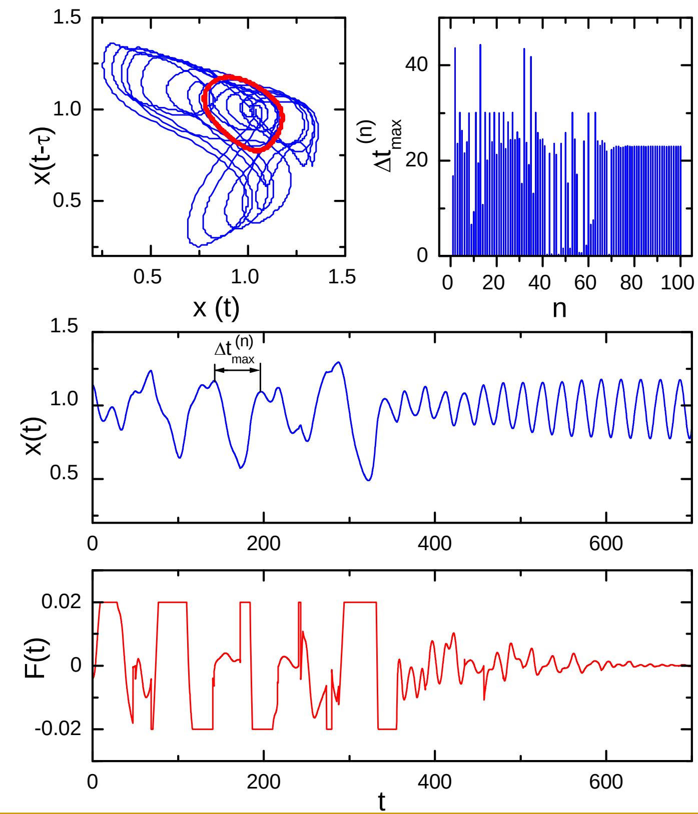
$$F_{n+1} = K[y(t) - y(t - \tau_n)]$$

$$F_n = K[y(t) - y(t - \tau_{n-1})]$$

$$n \to \infty \quad \tau_n \to T$$

Example: Mackey-Glass system

$$\frac{dy}{dt} = \frac{0.2y(t-\tau_M)}{1+y^{10}(t-\tau_M)} - 0.1y + F_n(t)$$

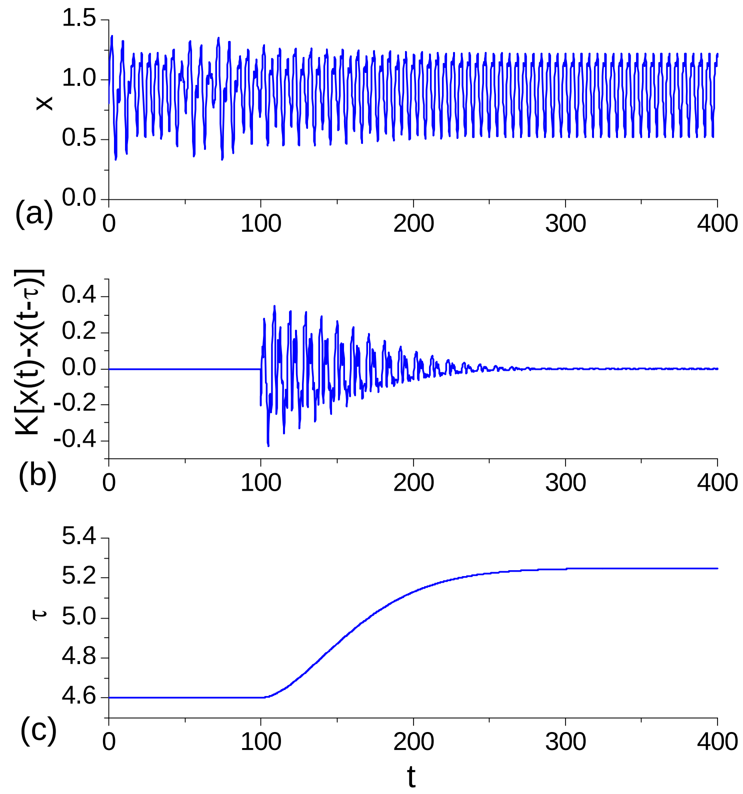


Adaptive DFC with a continuously varying time delay

V. Pyragas, K. Pyragas, PLA (2011)

Example: Mackey-Glass system

$$\dot{\vec{x}} = \vec{f}\left(x(t), K\left[s(t)-s(t-\tau(t))\right]\right),\ s(t)=g\left(\vec{x}(t)\right)$$
$$\dot{\tau} = -\beta G(t)$$
$$\dot{G} = \gamma\left[s(t)-s(t-\tau(t))\right]\left[s(t-\tau(t))-u(t)\right]-\nu G(t)$$
$$\dot{u} = \gamma\left[s(t-\tau)-u(t)\right]$$



The algorithm is derived using a gradient-descent method

Another adaptive algorithm based on the speed-gradient method → **Lehnert et al., Chaos (2011)**

The list of other DFC problems not discussed in this review

- **Control of oscillation coherence in noisy systems**
  - Goldobin, Rosenblum, Pikovsky, Physica A (2003); PRE (2003);
  - Janson, Balanov, Schoell, PRL (2004);
  - Balanov, Janson, Schoell, Physica D (2004);
  - Hauschildt, Janson, Balanov, Schoell. PRE (2006);
  - Pomplun, Amann, Schoell, EPL (2005) ; PRE (2007);
- **Control of synchronization in neural networks**
  - Rosenblum, Pikovsky, PRL (2004); PRE (2004);
  - Hauptmann, Popovych, Tass, Neurocomputing (2005); Biol. Cybern. (2005);
  - Popovych, Hauptmann, Tass, PRL (2005); Biol. Cybern. (2006); Int. J. Bif. Chaos (2006;
  - Popovych, Hauptmann, Tass, PRE (2010);
- **Control of spatio-temporal patterns in reaction-diffusion systems**
  - Baba, Amann, Schoell, Just PRL (2002);
  - Beck, Amann, Schoell, Socolar, Just, PRE (2002);
  - Balanov, Beato, Janson, Engel, Schoell PRE (2006);
  - Schneider, Schoell, Dahlem, Chaos (2009);
- **DFC based bifurcation analysis for experiments**
  - Sieber, Gonzalez-Buelga, Neild, Wagg, Krauskopf, PRL (2008);
- **DFC with the variable and distributed time delay**
  - Gjurchinovski, Urumov, EPL (2008);
  - Juengling, Gjurchinovski, Urumov, http://arxiv.org/pdf/1202.0519.pdf (2012)

Conclusions

The delayed feedback control is still active area of research

Hopefully, new interesting theoretical and experimental results will appear in the near future

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