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Adaptive modification of the delayed feedback control algorithm with a continuously varying time delay

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ABSTRACT

We propose a simple adaptive delayed feedback control algorithm for stabilization of unstable periodic orbits with unknown periods. The state dependent time delay is varied continuously towards the period of controlled orbit according to a gradient-descent method realized through three simple ordinary differential equations. We demonstrate the efficiency of the algorithm with the Rössler and Mackey–Glass chaotic systems. The stability of the controlled orbits is proven by computation of the Lyapunov exponents of linearized equations.

1. Introduction

Although delayed feedback control (DFC) algorithm has been introduced almost two decades ago [1] it is still one of the most active fields in applied nonlinear science [2]. This algorithm provides a simple, robust, and efficient tool for stabilization of unstable periodic orbits (UPOs) in nonlinear dynamical systems. The control signal in the DFC algorithm is formed from a difference between the current state of the system and the state of the system delayed by one period of a target orbit. Such control signal allows one to treat the controlled system as a black box; it does not require any exact knowledge of either the form of the periodic orbit or the system's equations. The method is asymptotically noninvasive because the control force vanishes whenever the target UPO is reached. The DFC algorithm has been successfully implemented in quite diverse experimental systems from different fields of science. Some details of experimental implementations as well as various modifications of the DFC algorithm can be found in the review paper [3]. In addition, we refer to recent theoretical and experimental results concerning the refuting of the odd number limitation of the DFC algorithm [4–7], the global properties of the DFC (the basins of attraction of the stabilized states) [8,9,6] and the DFC based bifurcation analysis for experiments [10,11]. An important practical application of the DFC algorithm has been recently demonstrated by Yamasue et al. [12]. By successful implementation of the DFC method in an atomic force microscope the authors managed to stabilize cantilever oscillation and remove artifacts on a surface image.

Experimental implementation of the DFC method requires a knowledge of delay time, which is equal to the period of actual UPO. However, for autonomous systems this period is not known a priori. Furthermore, in real experimental situation the period of actual UPO may evolve due to the evolution of the system parameters under the effects of exogenous, unpredictable factors. In this context, adaptive control techniques with an automatic adjustment of the delay time are desired.

The problem of estimating the period of a target UPO from observed experiential data has been considered in several publications. In the original paper [1], it has been shown that the amplitude of DFC perturbation has a resonance-type dependence on the delay time and the periods of UPOs can be extracted from the minima of this dependence. More sophisticated theoretical foundation for obtaining the UPO period from observed control signal has been developed in Ref. [13]. The first adaptive technique employing online variation of the delay time has been proposed in Ref. [14]. Here the delay time is adjusted in a discrete way according to the distance between successive maxima of the output signal. Another adaptive discrete-time technique based on the gradient-descent method has been proposed in Refs. [15–18]. In Ref. [19], the delay has been adapted in a continues way by using an information on the phase difference between the output and delayed output signals. The latter has been extracted with the help of a nonlinear filter known from phase-locked loops within analogue

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communication systems. This approach is rather complicated for experimental implementation. In Ref. [20], an adaptive controller with the delay time and control gain being both state-dependent have been proposed. The controller provides a monotonous increase of the control gain, while the theoretical foundation of this algorithm is based on the assumption that trajectories of the controlled system are bounded. For typical systems, this assumption is difficult to justify and to avoid the runaway of trajectories to infinity the authors have resorted to additional implements such as an impulsive switching of control perturbation and a truncation of the control force. As a result, the convergence of the delay time to the period of actual UPO has been attained only approximately.

Note the advantages of time-varying delay have been also discussed in the literature in the context of improving the stability properties of the DFC. Gjurchinovski and Urumov [21] have recently demonstrated that the modulation of the delay time in a fixed interval around a nominal value enlarges the stability domain of the DFC algorithm.

In this latter, we propose a simple adaptive continuous-time algorithm with a state dependent delay, which preserves the main advantages of the original DFC algorithm. The algorithm is based on resonance dependence of the DFC perturbation on the delay time pointed out in Ref. [1]. We define an appropriate potential that describes a running average of the square of control perturbation and minimize it via a gradient-descent method. This procedure is performed with three simple ordinary differential equations and does not require any complex online computations. The algorithm is noninvasive in the sense that it does not change the profile and the period of the inherent UPO and we are able to prove its validity by the linear stability analysis of the controlled UPO. The rest of the Letter is organized as follows. In Section 2 we present a general formulation of our adaptive DFC scheme and describe its main ideas. In Section 3 we demonstrate the efficiency of our algorithm for specific chaotic systems. The Letter is finished with the conclusions presented in Section 4.

2. General formulation of the adaptive DFC strategy

Let us assume we have a dynamical system or an experiment where the vector $\boldsymbol{X}(t)$ denotes the complete state of the system. Suppose that we do not know the state explicitly and are just able to measure a scalar signal $s(t) = g[\boldsymbol{X}(t)]$ that depends on the internal state through a function g. Let $\boldsymbol{\xi}(t) = \boldsymbol{\xi}(t-T)$ denote the periodic unstable state of a period T which we intend to stabilize. The key idea of the standard DFC algorithm consists in imposing a force on the system which is proportional to the time-delayed difference of the measured signal:

$$\Delta s(t) = s(t) - s(t-\tau) = g\big[\boldsymbol{X}(t)\big] - g\big[\boldsymbol{X}(t-\tau)\big]. \tag{1}$$

The equation of motion of DFC controlled system satisfying the above assumptions can be presented in the form:

$$\dot{\boldsymbol{X}}(t) = \boldsymbol{F}\big[\boldsymbol{X}(t), K\Delta s(t)\big]. \tag{2}$$

Here the first argument in the right-hand side shows the dependence of the vector field on internal degrees of freedom, while the second argument denotes the dependence on the control force $K\Delta s(t)$, where K is a control gain. The free ($K = 0$) system has an unstable periodic solution $\boldsymbol{X}(t) = \boldsymbol{\xi}(t) = \boldsymbol{\xi}(t-T)$. Provided that the delay time τ coincides with the period T of the orbit the control scheme for $K \neq 0$ satisfies the constraint that the force vanishes when the target state $\boldsymbol{\xi}(t)$ is reached, i.e., the method yields a noninvasive control scheme.

In this Letter, we suppose that the period T of UPO is unknown and our aim is to construct an adaptive algorithm that provides an automatic convergence of the delay τ to the period T. The adaptation should work even if the period T slowly varies in time, for example due to the evolution of the system parameters under the effects of exogenous factors. We intend to adapt the delay continuously. Thus the delay time τ in our algorithm becomes a state dependent variable and we seek to construct an appropriate dynamic equation for this variable.

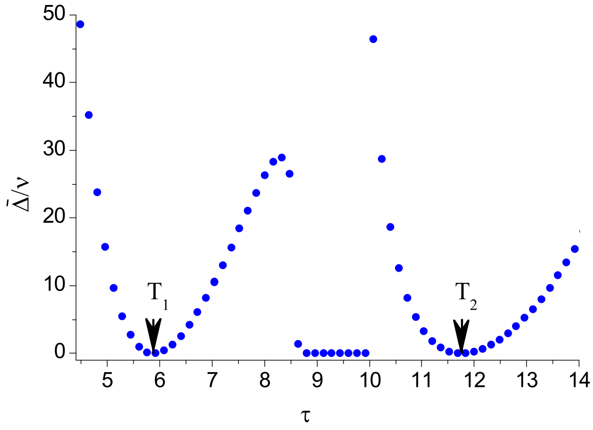

Fig. 1. The dependence of the mean squared exponentially weighted control signal $\bar{\Delta}$ on the delay τ for the Rössler system (14)–(16). The control is performed by the standard DFC algorithm assuming that τ in Eq. (3) is a time-independent parameter. The measured signal is $s(t) = y(t)$ and the DFC perturbation is applied to the second equation of the system (14)–(16). The parameters are: $a = 0.2$, $b = 0.2$, $c = 5.7$, $K = 0.2$, and $\nu = 0.01$. The minima at $\tau = T_1$ and $\tau = T_2$ correspond to the period-1 and period-2 UPOs, respectively. The small values of $\bar{\Delta}$ in the interval $\tau = [8.6, 9.9]$ are related to the stabilization of the steady state.

The idea of our algorithm is based on the observation that the mean square $\langle[\Delta s(t)]^2\rangle$ of the control signal $\Delta s(t)$ has a resonance-type dependence on the delay time, provided the control gain K is chosen appropriately [1]. The variable $\langle[\Delta s(t)]^2\rangle$ has a minimum reaching a zero value when τ coincides with the period T of UPO. Thus minimizing this variable via a gradient-descent method one can adapt the value of τ to the period T. In order to simplify online computations we follow the ideas of Ref. [22], and instead of $\langle[\Delta s(t)]^2\rangle$ we introduce a mean squared exponentially weighted control signal

$$\bar{\Delta}(t) = \int^{t} e^{-\nu(t-t')}\big[s(t') - s\big(t' - \tau(t')\big)\big]^2\,dt'. \tag{3}$$

Here ν^{-1} defines the width of the window $[t-\nu^{-1}, t]$ where the running average of the control signal is performed. As an example, in Fig. 1 we demonstrate the dependence of $\bar{\Delta}$ on the delay time τ for the Rössler system, whose equations are presented in the next section. In this demonstration, we suppose that τ is an independent of time parameter, then for sufficiently small ν the variable $\bar{\Delta}$ is also independent of time. The pronounced minima in the dependence $\bar{\Delta}$ on τ are indeed observed when τ coincides with the period-1 and period-2 UPOs embedded in the Rössler attractor.

Thus when stabilization of UPO is achieved ($\tau = T$), we have that $\bar{\Delta} = 0$ and $\bar{\Delta} > 0$ otherwise. Hence we can program τ to minimize $\bar{\Delta}$. The algorithm is greatly facilitated if we choose ν so that

$$\tau_s < \nu^{-1} < \tau_p \tag{4}$$

where τ_s is the time scale on which the system dynamics evolves [the time scale for the evolution of $s(t)$], and τ_p is the time scale on which the system parameters evolve [the time scale for the evolution of the period $T(t)$ of UPO]. With this assumption, $\tau(t')$ in Eq. (3) can be replaced by $\tau(t)$ to yield the following approximation to $\bar{\Delta}$:

$$\bar{\Delta}(t) \approx \Delta(t) = \int^{t} e^{-\nu(t-t')}\big[s(t') - s\big(t' - \tau(t)\big)\big]^2\, dt'. \tag{5}$$

This approximation means that we neglect the variation of τ within the running average window $[t - \nu^{-1}, t]$. Since $\Delta = 0$ at $\tau = T$ and is positive otherwise, one can program τ so as to seek the minimum of Δ by the following gradient descent relaxation,

$$\frac{d\tau}{dt} = -\beta \frac{d\Delta}{d\tau} \equiv -\beta G, \tag{6}$$

where $\beta^{-1} > \nu^{-1}$ is a parameter that determines the relaxation time scale and $\Delta(\tau)$ may be viewed as a potential function for the gradient flow (6). To estimate the gradient $G(t) = d\Delta/d\tau$ of the potential we differentiate Eq. (5) with respect to τ and obtain:

$$G(t) = 2\int^{t} e^{-\nu(t-t')}\big[s(t') - s\big(t' - \tau(t)\big)\big]\frac{d}{dt'} s\big(t' - \tau(t)\big)\, dt'. \tag{7}$$

To eliminate the need for calculating the integral (7), we use again the argument that $\tau(t)$ is a slowly varying function within the running average window $[t - \nu^{-1}, t]$ and replace back $\tau(t)$ by $\tau(t')$. Then the function $G(t)$ satisfies the differential equation:

$$\dot{G} = 2\big[s(t) - s\big(t - \tau(t)\big)\big]\dot{s}\big(t - \tau(t)\big) - \nu G. \tag{8}$$

In this equation we need the expression of derivative $\dot{s}(t - \tau(t))$. We estimate this derivative via a high-pass filter

$$\dot{u} = \gamma\big[s\big(t - \tau(t)\big) - u\big] \tag{9}$$

described by an additional dynamic variable $u(t)$. The time constant of the filter has to be less than the characteristic time scale of the variable $s(t)$, $\gamma^{-1} < \tau_s$. Then the derivative can be approximated as $\dot{s}(t - \tau(t)) \approx \gamma[s(t - \tau(t)) - u]$. Inserting this approximation into Eq. (8) and collecting Eqs. (2), (6), (8), and (9) we finally get a system of differential equations

$$\dot{\boldsymbol{X}} = \boldsymbol{F}\big(\boldsymbol{X}, K\big[s(t) - s(t - \tau)\big]\big), \tag{10}$$

$$\dot{\tau} = -\beta G, \tag{11}$$

$$\dot{G} = 2\gamma\big[s(t) - s(t - \tau)\big]\big[s(t - \tau) - u\big] - \nu G, \tag{12}$$

$$\dot{u} = \gamma\big[s(t - \tau) - u\big] \tag{13}$$

that describes our adaptive algorithm. We stress that Eqs. (10)–(13) satisfy the constraint that the control force vanishes when τ approaches T and the system reaches the target UPO, i.e., the described adaptive method yields a noninvasive control scheme as well as the original DFC algorithm.

When choosing the controller parameters in a real experiment the above mentioned inequalities $\gamma^{-1} < \tau_s < \nu^{-1} < \beta^{-1}$ must be kept in mind. More precise tuning of the parameters can be performed by trial and error taking the amplitude of the control force as a criterion of UPO stabilization. If the system equations and coupling of the control force are specified then the stability of controlled UPO can be verified explicitly. In the next section, we perform such an analysis for two specific systems.

Although we have initially assumed that the measured signal $s(t) = g[\boldsymbol{X}(t)]$ is a scalar variable, this assumption is not essential for our algorithm and has been made only to simplify the description. The algorithm admits a straightforward generalization for the case when the measured signal and control signal are vector variables.

3. Numerical demonstrations

3.1. Controlling the Rössler system

Now we apply our adaptive DFC algorithm to the chaotic Rössler system [23]. The state of the Rössler system is defined by 3d vector $\boldsymbol{X} = (x, y, z)$ and the first equation in system (10)–(13) takes the form:

$$\dot{x} = -\omega y - z, \tag{14}$$

$$\dot{y} = \omega x + ay - K\big[s(t) - s(t - \tau)\big], \tag{15}$$

$$\dot{z} = b + z(x - c). \tag{16}$$

Here we suppose that the measured signal is the y component, i.e., $s(t) = y(t)$ and the control force is applied only to the second equation of the Rössler system. The parameters of the Rössler system are chosen such ($\omega = 1$, $a = 0.2$, $b = 0.2$, $c = 5.7$) that without control ($K = 0$) it is in chaotic regime.

First we perform the linear stability analysis of the controlled Rössler system. Such an analysis is important for both the proof of validity of the proposed algorithm and the optimal choice of the controller parameters. In order to perform such an analysis we denote the solution of Eqs. (14)–(16), (11)–(13) corresponding to the target UPO by: $\boldsymbol{X} = \boldsymbol{\xi} = (\xi_x, \xi_y, \xi_z)$, $\tau = T$, $G = 0$, and $u = u_\xi$. Then the equations for small deviations from this solution are obtained via linearization of the controlled system about this solution using the substitution: $\boldsymbol{X} = \boldsymbol{\xi} + \delta\boldsymbol{X}$, $\tau = T + \delta\tau$, $G = \delta G$, and $u = u_\xi + \delta u$, where $\delta\boldsymbol{X} = (\delta x, \delta y, \delta z)$. For small deviations from the target state we get the system:

$$\delta\dot{x} = -\omega\delta y - \delta z, \tag{17}$$

$$\delta\dot{y} = \omega\delta x + a\delta y - K\big[\delta y - \delta y(t - T) + \dot{\xi}_y\delta\tau\big], \tag{18}$$

$$\delta\dot{z} = \xi_z\delta x + (\xi_x - c)\delta z, \tag{19}$$

$$\delta\dot{\tau} = -\beta\delta G, \tag{20}$$

$$\delta\dot{G} = 2\gamma(\xi_y - u_\xi)\big[\delta y - \delta y(t - T) + \dot{\xi}_y\delta\tau\big] - \nu\delta G, \tag{21}$$

$$\delta\dot{u} = \gamma\big[\delta y(t - T) - \delta u - \dot{\xi}_y\delta\tau\big]. \tag{22}$$

The distinct feature of these linearized equations is that the delay time in all time-delay terms is equal here to the exact period T of the target orbit and does not depend on the state of the system. Thus contrary to the nonlinear system (14)–(16), (11)–(13) that is described by delay-differential equations with the state-dependent delays, this system is much simpler, since it is defined by delay-differential equations with constant delays.

In Fig. 2 we present the results of linear stability analysis of the target state based on Eqs. (17)–(22). We show the three largest Lyapunov exponents of the period-1 and period-2 UPOs as functions of the control gain K. Since the system is autonomous there exists the zero exponent for any K. All other exponents are negative in an interval of the feedback gain $K_1 < K < K_2$. This interval defines the stability region of the target orbit, i.e., the values of the coupling gain where the adaptive algorithm works. In the stability interval, there exists an optimal value of the coupling gain K_0 where the largest non-zero exponent has a minimum. This value of the coupling gain provides the fastest convergence of nearby trajectories to the target state.

To confirm the results of the linear stability analysis we have numerically verified the performance of our algorithm in the conditions of real experimental situation, when no information on the UPO period and profile is known. The nonlinear system of delay-differential equations (14)–(16), (11)–(13) with the state dependent delay has been integrated using the DDESD program in MATLAB 7. Depending on the initial value of the delay time $\tau(0) = \tau_0$

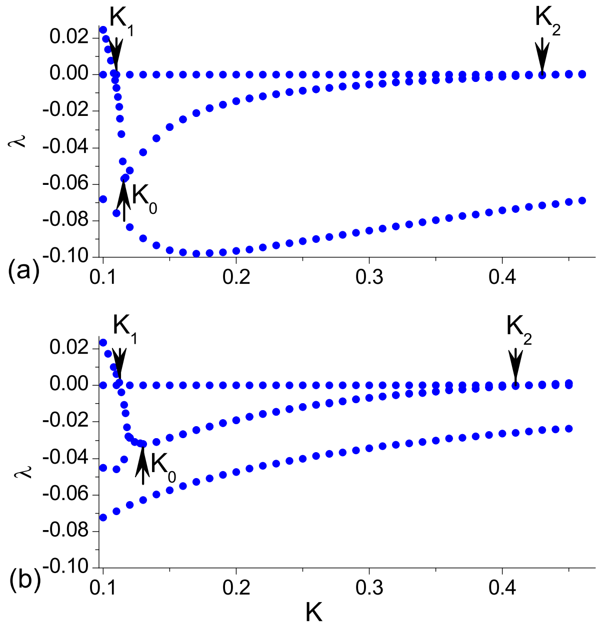

Fig. 2. The dependence of three largest Lyapunov exponents λ on the feedback gain K for the period-1 (a) and period-2 (b) UPOs of the Rössler system controlled by the adaptive DFC algorithm. The exponents are obtained from the linear system (17)–(22). The values of the controller parameters are: $\gamma = 5$, $\beta = 0.002$, and $\nu = 0.02$. The boundaries of the stability interval for the period-1 orbit are: $K_1 = 0.11$, $K_2 = 0.43$ and the optimal value of the control gain is $K_0 = 0.116$. For the period-2 orbit the corresponding parameters are: $K_1 = 0.12$, $K_2 = 0.41$, and $K_0 = 0.13$.

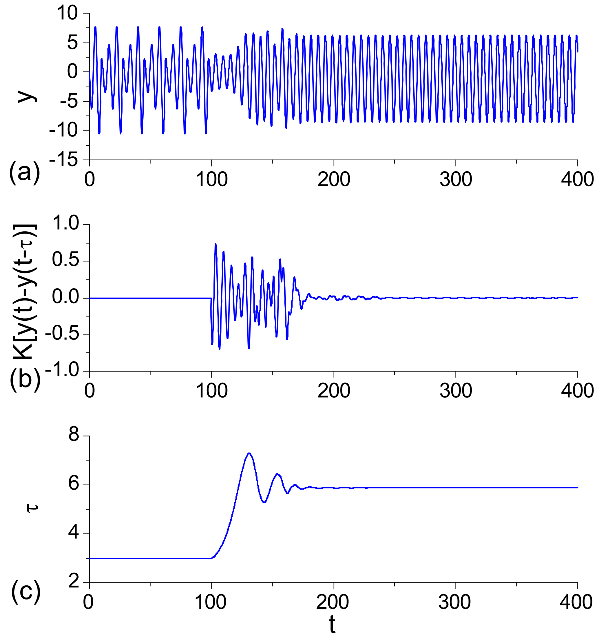

Fig. 3. Adaptive stabilization of the period-1 UPO of the Rössler system. Dynamics of the measured signal $s(t) = y(t)$ (a), control force $K[y(t) - y(t - \tau)]$ (b), and state-dependent delay time $\tau(t)$ (c) are obtained by numerical integration of the nonlinear system (14)–(16), (11)–(13). The controller parameters are: $\gamma = 5$, $\beta = 0.002$, and $\nu = 0.02$. The initial value of the delay time is $\tau_0 = 3$. For $t < t_c = 100.0$ the feedback force is switched off ($K = 0$), and for $t \geqslant t_c$ it is activated with an optimal value of the feedback gain $K = K_0 = 0.116$.

different UPOs have been stabilized. In Fig. 3 we demonstrate the stabilization of the period-1 UPO for the initial delay $\tau_0 = 3$, which differs considerably from the UPO period $T_1 \approx 5.88$. We see that the adaptive algorithm forces the variable delay to converge towards the period of UPO. When the delay time reaches the value equal to the UPO period the control force vanishes and the system persistently moves along the stabilized UPO. Here the numerical demonstration has been performed with an optimal value of the control gain $K = K_{op}$. However, we note that the algorithm works for any K taken from the stability interval $K \in (K_1, K_2)$.

If the initial value τ_0 is away from T_1 but closer to the period $T_2 \approx 11.76$ of the period-2 UPO, then the adaptive controller stabilizes the period-2 UPO. An example of stabilization of the period-2 UPO with the initial delay $\tau_0 = 10$ is demonstrated in Fig. 4. In addition, here we have taken into account a small noise to make our model as close as possible to a real experimental situation. White Gaussian noise terms $\varepsilon\eta_i(t)$ with $\langle\eta_i(t)\rangle = 0$ and $\langle\eta_i(t)\eta_j(t')\rangle = \delta_{ij}\delta(t - t')$ have been added to the r.h.s. of each equation of the Rössler system (14)–(16). Here the parameter ε governs the amplitude of noise, δ_{ij} is the Kronecker delta, $\delta(t)$ is the Dirac delta function and the indexes i and j numerate the equations of the Rössler system, i.e. the values of $i, j = 1, 2$ and 3 correspond to Eqs. (14), (15) and (16), respectively. In Fig. 4 the amplitude of noise is $\varepsilon = 0.01$. We see that the algorithm does work in the presence of noise; the delay time converges to an approximate value of the UPO period and the system approaches the controlled orbit. Now after the transient process, the control force does not vanish; it fluctuates around the zero level with a small amplitude proportional to the noise intensity ε. For sufficiently large noise amplitude $\varepsilon > 0.03$, the algorithm fails to stabilize the orbit.

To verify the adaptation abilities of the algorithm in the case of slowly varying system parameters, we have considered the control of the Rössler system with the time-varying parameter ω:

$$\omega = \omega_0 + \omega_1 \sin(2\pi t/T_\omega). \tag{23}$$

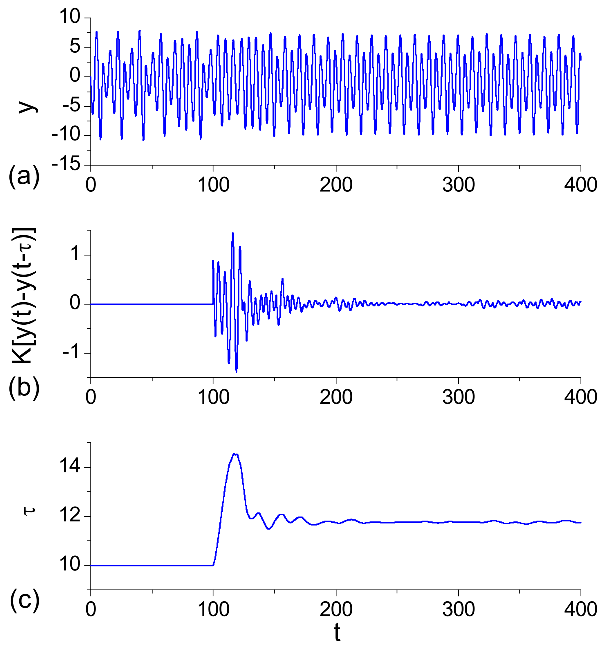

Fig. 4. Adaptive stabilization of the period-2 UPO of the Rössler system in the presence of noise with the amplitude $\varepsilon = 0.01$. The values of the parameters are the same as in Fig. 3 except of the value of the feedback gain and the initial value of the delay time, which here are $K = 0.125$ and $\tau_0 = 10$, respectively.

This parameter is responsible for the frequency of the Rössler attractor and its variation causes the variation of the periods of UPOs embedded in the strange attractor. In Fig. 5 we demonstrate the stabilization of the period-1 UPO when the parameter ω is varied by a sinus law with the amplitude $\omega_1 = 0.15$ and the period $T_\omega = 200$ around the mean value $\omega_0 = 1$. We see that the delay

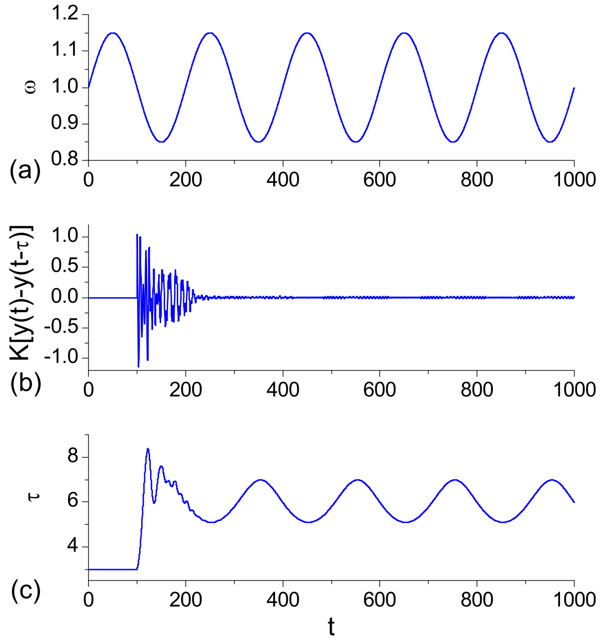

Fig. 5. Adaptive stabilization of the period-1 UPO of the Rössler system in the case of periodical variation of the parameter ω [Eq. (23), $\omega_0 = 1$, $\omega_1 = 0.15$, $T_\omega = 200$]. The controller parameters are the same as in Fig. 3. The control force with the strength $K = 0.12$ is switched on at the moment $t = 100$. The initial value of the delay time is $\tau_0 = 3$. The diagrams show the dynamics of the parameter ω (a), control force (b), and delay time (c).

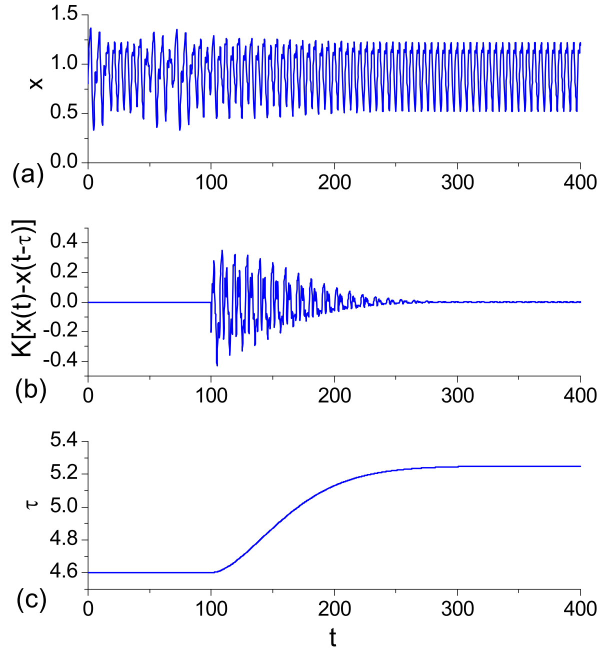

Fig. 6. Adaptive stabilization of the period-1 UPO of the MG system. Dynamics of the measured signal $s(t) = x(t)$ (a), control force $K[x(t) - x(t-\tau)]$ (b), and state-dependent delay time $\tau(t)$ (c) are obtained by numerical integration of the nonlinear system (24), (11)–(13). The controller parameters are: $\gamma = 5.0$, $\beta = 0.005$, and $\nu = 0.05$. The initial value of the delay time is $\tau_0 = 4.6$. The control force with an optimal feedback strength $K = K_0 = 0.33$ is switched at the moment $t = 100$.

time successfully adapts to the varying period of UPO such that the control force remains small during these variations. The algorithm works even when the period of UPO varies in a rather large interval (up to 30% of the mean value).

3.2. *Controlling the Mackey–Glass system*

Now we demonstrate the efficiency of our algorithm for a more complex dynamical system described by a delay-differential equation. We consider the control of delay induced chaotic behavior in the Mackey–Glass (MG) [24] system: $\dot{x} = ax(t-\Theta)/[1 + x^b(t-\Theta)] - cx$. This system has been initially introduced as a model of blood generation for patients with leukemia. Later this model became popular in chaos theory as a model for producing high-dimensional chaos to test various methods of chaotic time-series analysis, controlling chaos, etc. The electronic analog of this system has been proposed in Ref. [25].

Although the MG system is defined by a scalar variable $x(t)$, its dynamics is characterized by an infinite-dimensional phase space because of the intrinsic delay Θ. We choose the parameters of the MG system such ($a = 2.0$, $b = 10.0$, $c = 1.0$, $\Theta = 2.0$) that it displays a chaotic behavior. In order to stabilize an UPO embedded in a chaotic attractor we apply our adaptive algorithm (10)–(13). Then the first equation in system (10)–(13) takes the form:

$$\dot{x} = \frac{ax(t-\Theta)}{1 + x^b(t-\Theta)} - cx - K\big[s(t) - s(t-\tau)\big]. \qquad (24)$$

Here the measured signal is $s(t) = x(t)$. Note, that unlike to the previous example the controlled MG system (24), (11)–(13) contains two delays, namely, the constant intrinsic delay Θ and the state-dependent delay τ of the control loop. We intend to stabilize the period-1 UPO of the MG system with the period $T \approx 5.249$.

The linear stability analysis preformed for the MG system shows similar results as for the Rössler system. Here the UPO is stable in the interval of the feedback gain $[K_1, K_2] = [0.28, 4.48]$ and the optimal value is $K_0 = 0.33$. An example of a nonlinear analysis based on Eqs. (24), (11)–(13) is presented in Fig. 6. An adaptive stabilization of the period-1 UPO of the MG system is attained with the initial value of the delay time $\tau_0 = 4.6$, which differs considerably from the UPO period $T = 5.249$. Thus the algorithm enables the control of infinite-dimensional systems described by delay-differential equations.

4. Conclusions

We have proposed a simple adaptive delayed feedback control algorithm for the stabilization of unstable periodic orbits with unknown periods. The algorithm provides an automatic convergence of the state-dependent delay time to the period of the target orbit and can be used for the tracking problems in systems with slowly varying parameters. Unlike to the previous adaptive algorithms [14–18] based on discrete-time variation of the delay time, here the delay time is adapted continuously. This makes the algorithm attractive for experimental implementation by means of analogue electronics, which is superior for fast dynamical systems. The algorithm is based on the descent gradient method embodied by three simple ordinary differential equations. Note, that the discrete-time versions of the descent gradient method proposed in Refs. [15–17] imply the use of a computer, since the potential functions, which are minimized in these algorithms, are expressed in the form of complex integrals. In our algorithm, we have avoided the integral expressions. Thus we have preserved the main advantages of the original DFC method — the non-invasiveness and the ability of implementation without resort to a computer. An additional advantage of our algorithm is that it does not require a knowledge of the period of controlled orbit.

Theoretically, our algorithm provides the exact convergence of the delay time to the period of unstable orbit and we do not need any additional implements like impulsive switching of control perturbation or truncation of the control force used in Ref. [20]. Our adaptive algorithm stabilizes the controlled orbit without changing its profile. The linear stability of orbits under action of adaptive control force is proven by computation of Lyapunov exponents.

We have demonstrated that our algorithm works in the presence of noise, which confirms its operational capability in the real experimental situation. The examples presented in this Letter show that the algorithm is efficient for both the finite-dimensional systems described by ordinary differential equation and the infinite-dimensional systems defined by delay differential equations. The adaptive algorithm is of particular interest for the latter class of systems. Because of the infinite-dimensional phase space, the information on the period of unstable orbits is difficult to extract for such systems even if the model is known. Our algorithm does not require such an information and finds the period of controlled orbit automatically in the process of stabilization.

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